Consider the $\mathcal{R}$ classes ($R_1, R_2, R_3$ and $R_7$)of the semigroup $S$ for $n = 3$

$R_1 = \{(12 -), (23 -), \}$,

$R_2 = \{(1 - 2),\ (2 - 3), \}$,

$R_3 = \{(-12),\ (-23), \}$ and

$R_4 = \{(1 - -), (2 - -), (3 - -)$,

$R_5 = \{(-1 -), (-2 -), (-3 -), \}$,

$R_6 = \{(- - 1), (- - 2), (- - 3)\}, R_7 = \{(- - -)\}$

### *D. IDENTITY DIFFERENCE PARTIAL-ORDER PRESERVING TRANSFORMATION SEMIGROUP $IDPO_n$*

$R_1 = \{(112), (223)\},\ R_2 = \{(122), (233)\}$,

$R_3 = \{(12 -), (23 -)\}, R_4 = \{(-12), (-23)\}$,

$R_5 = \{(1 - 2), (2 - 3)\}, R_6 = \{(111), (222), (333)\}$,

$R_7 = \{(11 -), (22 -), (33 -)\}$,

$R_8 = \{(-11), (-22), (-33)\}$,

$R_9 = \{(1 - 1), (2 - 2), (3 - 3)\}$,

$R_{10} = \{(1 - -), (2 - -), (3 - -)\}$,

$R_{11} = \{(-1 -), (-2 -), (-3 -)\}$,

$R_{12} = \{(- - 1), (- - 2), (- - 3)\}\ R_{13} = \{(- - -)\}$

are the elements of $IDPO_3$ in their respective $\mathcal{R}$ classes. $R_6, R_7, \ . \ . \ . \ R_{13}$ Contains the elements with the identity, constant and empty maps. After some mathematical operations we obtain the generating set by picking an element randomly from each $\mathcal{R}$ classes ($R_1,\ R_2, . \ . \ . \ R_5$). Generally, the minimum generating set $A$ for $S$ is of the sequence 5,17,49,129, . . . $\forall\, n \geq 3,\ R_1, R_2, R_3,\ R_4,\ . \ . \ . \ R_{13} \in \mathcal{R}$.

## IV. THE RANKS OF IDENTITY DIFFERENCE TRANSFORMATION SEMIGROUP

In this section we shall give the theoretical and combinatorial proves for the ranks of $IDT_n$, $IDO_n$, $IDI_n$, $IDPOI_n$ and $IDPO_n$ respectively.

### *A. RANK OF IDENTITY DIFFERENCE FULL TRANSFORMATION SEMIGROUP $IDT_n$*

**THEOREM 1**

Let $\alpha \in S$ and $R(S)$ denote the rank of $S$. $\alpha \in R(S)$ iff $\alpha \in A$ is the generating set of $S$.

Proof

Let $S$ be the identity difference transformation semigroup. Suppose $(\alpha, \beta), (a, b), (c, d), \ . \ . \ . \in \mathcal{R}$ of $S$. Then $(\alpha, b, d, \ . \ . \ .) \in A$ if$(\alpha, b, d, \ . \ . \ .)\backslash\varsigma$, (where $\varsigma$ is a constant map elements). Suppose also that the choice of picking $\alpha, b, d, \ . \ . \ .$ in $(\alpha, \beta), (a, b), (c, d), \ . \ . \ .$ of $S$ is done randomly, then the products $(\alpha.b), (b.\alpha), (b.\alpha)d, \ . \ . \ .$ must generate the set of elements in $S$. Hence $\{\alpha, b, d, \ldots) \in A$ is the generating set for $S$ since one element is selected at random from each $\mathcal{R}$ which excludes the $\mathcal{R}$ classes containing the constant or identity maps.

In contradiction, Since $(\alpha, \beta), (a, b), (c, d), \ . \ . \ . \in \mathcal{R}$ of $S$. If the choice of selecting the elements of $A$ from different sets of $\mathcal{R}$ are random and $(\alpha, \beta, a, b)$ are selected for $A$, then $A \nsubseteq S$. Hence, $A$ is not a generating set for $S$ since the choice of elements does not consider the above stated conditions.

Thus, $\alpha \in R(S)$ iff $\alpha \in A$ and $A \subseteq S$. □

**THEOREM 2**

Let$S = IDT_n$. If $|\mathcal{R}|$ be the order of $\mathcal{R}$ in $S$. Then Rank $(S)$ is $(|\mathcal{R}| - 1) = 2^{(n-1)} - 1$.

Proof

Let $uS, \ldots n + (n + 1)(2^n - 2)S$ be the set of right ideals in $S\ \forall\, u \in S$. If $aS = bS = cS = \cdots nS$ ($\forall\, a, b, c, \ldots, n \in S$); then the elements in the set $\{a, b, c, \ldots, n\}$ are in same $\mathcal{R}$.

Suppose $a, b, c, \ldots, n$ are set of idempotent elements, then $|\text{im}\varepsilon|$ =1 where $\varepsilon$ is a constant map. Hence $a, b, c, \ldots, n\ \varepsilon\ E(S)$.

Let $uS = vS = \cdots 2(n - 1)S$ be set of equal right ideals in $S$ where $\{p, q, \ldots 2(n - 1)\}$, $\{m, n, \ldots 2(n - 1)\}, \ldots\ |\mathcal{R}| \setminus \mathcal{R}_c$ are different sets of $\mathcal{R}$ in S. Then $p, q, \ldots, 2(n - 1)$ (respectively $m, n, \ldots\ 2(n - 1)$) are elements in the same $\mathcal{R}$ where $\mathcal{R}_c$ is a set of $\mathcal{R}$ containing the constant map.

Suppose that $|\mathcal{R}|$ in $S$ contain the set $\mathcal{R}_c$ such that $\forall\, \alpha \in S\ \alpha(X_1\ . \ . \ . X_n) = i, i \geq 1$ is a constant map. Then the $Rank(S) = Min\{|A|: A \subseteq S, < A > = S\}$ is $|\mathcal{R}| - |\mathcal{R}_c| = |\mathcal{R}| - 1$.

Without loss of generality, for $n \geq 3$, $(aS = bS = \cdots = nS), (pS = qS = \cdots = 2(n - 1), (mS = nS = \cdots = 2(n - 1), \ldots 2^{(n-1)}$ implying that there are $2^{(n-1)}$ $\mathcal{R}$ in S.

Hence the $Rank(S) = |\mathcal{R}| - 1 = 2^{(n-1)} - 1 = 3,7,15,31,63, \ldots \forall\, n \geq 3$. □

See the table below;

| $n \geq 3$ | $Rank = (\|\mathcal{R}\| - 1) = 2^{(n-1)} - 1)$ |
|---|---|
| 3 | 3 |
| 4 | 7 |
| 5 | 15 |
| 6 | 31 |
| 7 | 63 |
| 8 | 127 |
| 9 | 255 |
| 10 | 511 |

**THEOREM 3**

Let S be identity difference full transformation semigroup. The

$$Rank(S) = \frac{1}{2}\sum_{p=0}^{n}\binom{n}{p} - \binom{n}{n} = 2^{(n-1)} - 1$$

Proof

Let $R(S) = \frac{1}{2}\sum_{P=0}^{n}\binom{n}{p} - \binom{n}{n}$

Implying that there are n ways P (respectively n) can be represented since $\binom{n}{p}$ and $\binom{n}{n}$.

Recall the identity

$\sum_{p=0}^{n}\binom{n}{p} = \binom{n}{0} + \binom{n}{1} + \binom{n}{2} + \binom{n}{3} + \cdots + \binom{n}{n-2} + \cdots + \binom{n}{n} = 2^n$. So that,

$\frac{1}{2}\sum_{p=0}^{n}\binom{n}{p} = \sum_{p=0}^{n}\frac{1}{2}\binom{n}{p} = \frac{1}{2}\left[\binom{n}{o} + \binom{n}{1} + \binom{n}{2} + \cdots + \binom{n}{n-2} + \cdots + \binom{n}{n}\right] = \frac{1}{2}[2^n] = 2^{-1}.2^n = 2^{n-1}$

Therefore $\frac{1}{2}\sum_{p=0}^{n}\binom{n}{p} - \binom{n}{n} = 2^{-1}.2^n - 2^0 = 2^{n-1} - 1$ .

□

*B. RANK OF IDENTITY DIFFERENCE ORDER PRESERVING TRANSFORMATION SEMIGROUP $IDO_n$*

Following the same approach as in 4.1 above, we obtain that $Rank(S) = (n-1) \forall\, n \geq 3$.

Lemma 1

Let $S$ be $IDT_n$ and $IDO_n$. If $nR_{classes}$ is the number of set of $R-classes$ in $S$. Then $Rank(S)$ is defined by the rule $(nR_{classes} - 1)$.

**THEOREM 4**

Let $S = IDO_n$. If $|\mathcal{R}|$ be the order of $\mathcal{R}$ in S. Then Rank $(S)$ is $(|\mathcal{R}| - 1) = (n-1)$ □

Since $Rank(S) = (n-1)\ \forall\, n \geq 3$ then,

| $n \geq 3$ | $Rank = (n-1)$ |
|---|---|
| 3 | 2 |
| 4 | 3 |
| 5 | 4 |
| 6 | 5 |
| 7 | 6 |
| 8 | 7 |
| 9 | 8 |
| 10 | 9 |

□

**THEOREM 5**

$Rank(S) = \binom{n-1}{n-2} = (n-1)$.

Proof

The expression $\binom{n-1}{n-2}$ is true that (n-1) can be represented in (n-2) ways and as such we have,

$\binom{n-1}{n-2} = \frac{(n-1)!}{[(n-1)-(n-2)]!(n-2)!} = \frac{(n-1)!}{(n-2)!.1!} = \frac{(n-1)!(n-2)!}{(n-1)!} = (n-1)$

Hence $\binom{n-1}{n-2} = (n-1)$ □

*C. RANK OF IDENTITY DIFFERENCE SYMMETRIC INVERSE TRANSFORMATION SEMIGROUP $IDI_n$*

**THEOREM 6**

Let $IDI_n = S$ be the identity difference symmetric inverse transformation semigroup. If $|\mathcal{R}|$ be the order of the set of R-classes in S then $Rank\ (S) = \left(|\mathcal{R}| - (n+1)\right) = (n-1) + \sum_{i=1}^{n-2}(i)$

Proof

Let $\alpha_1 S, \alpha_2 S, \ldots \alpha_{n+1+n^2(n-1)} S$ be the right ideals of $S\ \forall\ \alpha_1, \alpha_2, \ldots\ \alpha_{(n+1)+n^2(n-1)} \in S\ \forall n \geq 3$.

Where $\alpha_1 = (\begin{pmatrix} x_1 & x_2 \\ i & i+1 \end{pmatrix} \ldots\ x_n), \alpha_2 = \begin{pmatrix} x_1 & x_2 & \ldots & x_n \\ i+1 & i & \ldots & - \end{pmatrix}, \alpha_3 = \begin{pmatrix} x_1 & x_2 & \ldots & x_n \\ - & i & \ldots & i+1 \end{pmatrix}, \alpha_4 = \begin{pmatrix} x_1 & x_2 & \ldots & x_n \\ - & i+1 & \ldots & i \end{pmatrix}, \ldots \alpha_{t-1} = \begin{pmatrix} x_1 & \ldots & x_{n-1} & x_n \\ - & \ldots & i & i+1 \end{pmatrix}, \alpha_t = \begin{pmatrix} x_1 & \ldots & x_{n-1} & x_n \\ - & \ldots & i+1 & i \end{pmatrix}$.

Observe that, for all elements from $\alpha_{1,\ldots,}\alpha_t \in S$ there are precisely two point of maps from the domain to the codomain and every other point in each element are empty. As such, $\{\alpha_1, \alpha_2, \ldots\} \in S, \{\alpha_{3,}\alpha_4, \ldots\} \in S, \ldots, \{\alpha_{t-1,}\alpha_t \ldots\} \in S$ since, $\alpha_1 S = \alpha_2 S = \cdots, \alpha_3 S = \alpha_4 S = \cdots, \alpha_{t-1} S = \alpha_t S = \cdots$ .
Similarly. Let $\beta_{1,}\beta_{2,} \ldots, \beta_t \epsilon\ S$ such that, $\beta_1 = \begin{pmatrix} x_1 & x_2 & \ldots & x_n \\ i & - & \ldots & - \end{pmatrix}, \beta_2 = \begin{pmatrix} x_1 & x_2 & \ldots & x_n \\ i+1 & - & \ldots & - \end{pmatrix} \ldots, \beta_{t-1}\begin{pmatrix} x_1 & \ldots & x_{n-1} & x_n \\ - & \ldots & - & i \end{pmatrix}, \ldots$
$\beta_t\begin{pmatrix} x_1 & \ldots & x_{n-1} & x_n \\ - & \ldots & - & i+1 \end{pmatrix} \ldots$

If $\beta_1 S = \beta_2 S \ldots, \beta_{t-1} S = \beta_t S \ldots, then\ \{\beta_{1,}\beta_2 \ldots\}\epsilon\mathcal{R} \ldots \{\beta_{t-1}, \beta_t \ldots\}\epsilon\ \mathcal{R}\ in\ S.$

Recall that in S, there exist the empty map $\xi$ such that, $\xi\left(x_{1,}x_2 \ldots x_n\right) = \xi$, that is $\xi(x_1) = \xi, \xi(x_2) = \xi, \ldots, \xi(x_n) = \xi$. Hence $\{\xi\}$ is a set in $\mathcal{R}$

Therefore, $|\mathcal{R}| = (\{\alpha_1, \alpha_2, \ldots, \alpha_{2(n-1)}\}, \{\alpha_3, \alpha_4, \ldots, \alpha_{2(n-1)}\}, \ldots \{\alpha_{t-1}, \alpha_t, \ldots, \alpha_{2(n-1)}\}$ $, \ldots \{\beta_{1,}\beta_2, \ldots, \beta_n\}, \ldots \{\beta_{t-1}, \beta_{t1} \ldots \beta_n\}, \ldots, \xi$, Since by definition, Rank (S)=$Min\{|A|: A \subseteq S, < A > = S\}$, observe that the generating set is obtained by picking an element from each set of $\mathcal{R}$ (say $\alpha_{1,}\alpha_4, \alpha_6, \alpha_t, \ldots, \alpha_{(n-1)+\sum_{p=1}^{n-2} p} = A$) whose element must contain precisely two points of mappings and every other points empty map. Also, the choice of picking the elements of $A$ is of high importance since $< A > = S$.

Without loss of generality, in $\mathcal{R}(S)$

$|\mathcal{R}_1| =$

$[\{\alpha_1, \alpha_{2,} \dots \alpha_{2(n-1)}\}, \{\alpha_3, \alpha_4, \dots, \alpha_{2(n-1)}\}, \dots \{\alpha_{t-1}, \alpha_t, \dots \alpha_{2(n-1)}\} \dots] =$ $(n-1) + \sum_{i=1}^{n-2}(i)$ cases

$|\mathcal{R}_2| = [\{\beta_1, \beta_2, \dots, \beta_n\}, \{\beta_{t-1}, \beta_t, \dots, \beta_n\}] = n$ number of cases

$|\mathcal{R}_3| = \{\xi\} = 1$ case.

As such the Rank(S)= $[|\mathcal{R}| - (n+1)] = (n-1) + \sum_{i=1}^{n-2}(i)$

See the table below,

| $n \geq 3$ | $\lvert\mathcal{R}\rvert$ | $Rank(S) = (n-1) + \sum_{i=1}^{n-2} i$ |
|---|---|---|
| 3 | 7 | 3 |
| 4 | 11 | 6 |
| 5 | 16 | 10 |
| 6 | 22 | 15 |
| 7 | 29 | 21 |
| 8 | 37 | 28 |
| 9 | 46 | 36 |
| 10 | 56 | 45 |

**THEOREM 7**

$Rank(S) = \binom{n-1}{n-2} + \sum_{(i=1)}^{(n-2)} \binom{i}{i-1}$

Proof

$= \frac{(n-1)!}{[(n-1)-(n-2)]!(n-2)} + \sum_{i=1}^{(n-2)}[\frac{i!}{[i-(i-1)]!(i-1)!}] \quad = \frac{(n-1)!}{(n-2)!} + \sum_{i=1}^{n-2}(\frac{i!}{(i-1)!}) = \frac{(n-1)(n-2)!}{(n-2)!} + \sum_{i=1}^{n-2}(\frac{i(i-1)!}{(i-1)!})$

$= (n-1) + \sum_{i=1}^{n-2}(i)$. Therefore, $Rank(S) = (n-1) + \sum_{i=1}^{n-2}(i)$ □

*D. RANK OF IDENTITY DIFFERENCE PARTIAL ORDER SYMETRIC INVERSE TRANSFORMATION SEMIGROUP $IDPOI_n$*

**THEOREM 8**

Let $S = IDPOI_n$. Then

$Rank(S) = \binom{n-1}{n-2} + \sum_{(i=1)}^{(n-2)} \binom{i}{i-1} = (n-1) + \sum_{i=1}^{n-2}(i)$

□

**THEOREM 9**

Let $IDPOI_n = S$. If $|\mathcal{R}_c|$ be the order of the set of R-classes in S then

$Rank\ (S) = (|\mathcal{R}| - (n+1)) = (n-1) + \sum_{i=1}^{n-2}(i)$ (See theorem 6). □

*E. RANK OF IDENTITY DIFFERENCE PARTIAL ORDER PRESERVING TRANSFORMATION SEMIGROUP $IDPO_n$.*

**THEOREM 10**

Let $S = IDPO_n$. Then $Rank(S) = \left[\frac{1}{2}\sum_{p=0}^{n}\binom{n}{p}\right]\binom{n-2}{n-3} + \binom{n}{n} = 2^{n-1}(n-2) + 1$

Proof

$Rank(S) = \left[\frac{1}{2}\sum_{p=0}^{n}\binom{n}{p}\right]\binom{n-2}{n-3} + \binom{n}{n} = \frac{1}{2}\left[\binom{n}{o} + \binom{n}{1} + \binom{n}{2} + \dots + \binom{n}{n-2} + \dots + \binom{n}{n}\right]$ $(\frac{(n-2)!}{[(n-2)-(n-3)]!(n-3)!}) + \left[\frac{n!}{(n-n)!n!}\right] = \frac{1}{2}\left[\binom{n}{o} + \binom{n}{1} + \binom{n}{2} + \dots + \binom{n}{n-2} + \dots + \binom{n}{n}\right]\left(\frac{(n-2)(n-3)!}{(n-3)!}\right) + 1$

$= \frac{1}{2}.2^n(n-2) + 1 = 2^{n-1}(n-2) + 1.$

Therefore, $Rank(S) = \left[\frac{1}{2}\sum_{p=0}^{n}\binom{n}{p}\right]\binom{n-2}{n-3} + \binom{n}{n} = 2^{n-1}(n-2) + 1.$ □

**THEOREM 11**

Let $IDPO_n = S$ be the identity difference partial order preserving transformation semigroup.

Rank(S) = $|\mathcal{R}| - 2^n = (n-2)2^{(n-1)} + 1$.

Proof

Let $\alpha_1 S, \dots \alpha_{2^n + (2^{n-1})(n^2-n)} S$ $(where\ |S| = 2^n + (2^{n-1})(n^2-n))$, and $\alpha_1, \alpha_2, \dots, \alpha_{2^n + (2^{n-1})(n^2-n)}$ are arbitrary elements of S. Suppose there are $n.2^{(n-1)} + 1$ *set of* $\mathcal{R}$ in $S\ \forall\ n \geq 3$, then the sets must contain the partial identity maps say $\alpha_1 = \binom{\{X_1\ X_2\}\ \dots\ X_n}{i\ \ \ \dots\ -}, \alpha_2 = \binom{\{X_1\ X_2\}\ \dots\ X_n}{i+1\ \ \dots\ -}, \dots \alpha_n = \binom{\{X_1\ X_n\ \dots\ X_n}{n\ \ \dots\ -}$, the identity maps say $\beta_1 = \binom{\{X_{1\dots}\ X_n\}}{i} \dots \beta_n \binom{\{X_1, \dots X_n\}}{n}$ *and the empty map* $\xi = \binom{\{X_{1\dots}\ X_n\}}{-}$ where $\{\alpha_1, \alpha_2, \dots \alpha_n\}, \{\beta_1, \beta_2, \dots, \beta_n\}, \ . \ . \ . \ , \{\xi\} \in \mathcal{R}$.

Let $|\emptyset|$ be the order of the set of $\mathcal{R}$ containing the (partial) identity and the empty map elements, then by combinatorial analysis, $|\emptyset| = 2^n$.

Since in $\mathcal{S}$, the $|\ \mathcal{R}\ |$=$n.2^{(n-1)} + 1$ and $|\emptyset| = 2^n$. Then

$Rank(S) = |\mathcal{R}| - |\emptyset| = (n.2^{(n-1)} + 1) - 2^n = n.2^n.2^{-1} + 1 - 2^n = n.2^n.2^{-1} - 2^n + 1 = 2^n(n.2^{-1} - 1) + 1 = 2^n\left(\frac{n}{2} - 1\right) + 1 = 2^n\left(\frac{n-2}{2}\right) + 1 \quad = 2^n.2^{-1}(n-2) + 1 = 2^{(n-1)}(n-2) + 1.$

See table below;

| $n \geq 3$ | $\lvert\mathcal{R}\rvert = n.2^{n-1} + 1$ | $Rank(S) = 2^{(n-1)}(n-2) + 1$ | $\lvert Rank - \lvert\mathcal{R}\rvert\rvert = 2^n$ |
|---|---|---|---|
| 3 | 13 | 5 | 8 |
| 4 | 33 | 17 | 16 |
| 5 | 81 | 49 | 32 |
| 6 | 193 | 129 | 64 |
| 7 | 449 | 321 | 128 |
| 8 | 1025 | 769 | 256 |
| 9 | 2305 | 1793 | 512 |
| 10 | 5121 | 4097 | 1024 |

## V. V. CONCLUSION

We therefore conclude that the rank of identity difference transformation semigroup exist and can be easily obtained using the $R - classes$ of the respective subsemigroups as shown in section 3 and 4.

**Conflict of Interest Statement:**
This is to affirm that there is no conflict of interest amongst the authors or whosoever.

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