# Optimization model of the shell capsules geometry for a system for diagnosing damage to gas turbine blades in non-stationary

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**Abstract—Scientific research is devoted to the problem of diagnosing damage in the blades of gas turbine engines in non-stationary conditions. The paper proposes a model for a more accurate calculation of the parameters of the crack detection system using a system of capsules, inside of which, under the influence of pressure, there is a substance exhibiting ionizing properties. In the study, the relations for determining the uneven pressure due to the action of centrifugal tensile forces during the rotation of the turbine blade are obtained. The relations for calculating the minimum required diameter of the capsule to ensure the rupture of the capsule when cracks are opened for the effective operation of the crack detection system are obtained. Calculations of the pressure inside the capsule and the geometry of the capsule at different speeds of rotation of the turbine blade are carried out. An estimate of the error in calculating the pressure inside the capsule is given in the case of not taking into account the action of centrifugal forces. The obtained dependences will significantly optimize the system for detecting damage to turbine blades, and increase the efficiency and safety of the operation of gas turbine engines.**

**Keywords — centrifugal forces, damage detection system, pressure, turbine blade.**

## I. Introduction

In recent years, the modern development of turbomachinery has been directed towards improving the operational characteristics of gas turbine engines and optimizing the operation of their main elements. Turbine rotor blades are one of the most loaded elements today. Accordingly, there is an urgent problem associated with the detection of cracks and defects in the turbine blades in the engine operation mode. The main problem is that most of the methods for detecting damage to turbine blades are used in stationary conditions with the engine not running. However, it is dangerous to tear off part of the blade during the active development of a crack, which may not be detected during examination and monitoring of the blades in stationary conditions.

A large number of scientific works have been carried out in the field of designing turbine blades with thermal protection systems, intensification of heat exchange, and optimization of gas cooling. The thermal state of the turbine blade plays an important role in the ability to resist cracking since the long-term strength of the blade material decreases with prolonged high-temperature exposure. The problem of optimal cooling and the development of heat sink intensification systems are discussed in detail in [1], [2].

The forecast of the service life of the turbine blade depends not only on the power and thermal loading conditions but also on the material from which the blade is made. The use of composite materials in the manufacture of turbine blades, the issues of optimal design of the blade structure, assessment of damage to blades made of composite materials are considered in [3], [4]. Nickel is one of the main components of alloys used as materials for turbine blades because it has the best heat-resistant characteristics. The issues of forecasting the operation of nickel blades are considered in [5]. The use of other materials for turbine blades is considered in [6].

An important issue is related to the assessment of the effect of material fatigue on the process of crack formation since the turbine blades are in conditions of irregular force action. The calculation of blade damage under variable loading conditions was investigated in [7]. The features of fatigue failure according to the results of the experiment, and the use of methods of mechanical analysis are considered in [8], [9]. An experimental study of the destruction of elements of turbomachines and experiments on the propagation of cracks in the blades in the presence of vibrations were carried out in [10], [11].

Dynamic analysis of turbine blades is important in non-stationary conditions. In this case, the stresses and strains in the blade are functions of time, [12]. Evaluation of the condition of the blades of turbomachines is often carried out using statistical analysis methods. Monitoring the state of the turbine blade using a sensor network, and diagnostics of failures using neural and Bayesian networks are considered in [13], [14], [15]. A finite element method is an effective tool for modeling rotating blades with cracks, [16]. Computer analysis of turbine blades and analysis of turbine failures of aircraft engines was carried out in [17], [18].

The following stationary methods for assessing the state of turbine blades can be distinguished: ultrasonic method and magnetic flaw detection method, [19], photographic method for detecting defects in the turbine blade, [20], acoustic emission method for searching for damage to turbine blades, [21], method of vibration analysis and tracking the resonance of vibrations, [22], [23], crack detection using eddy current sensors, [24].

Comparing these methods, it is possible to identify their shortcomings. The imposition and influence of extraneous signals due to the operation of the engine are possible when using the acoustic emission method and the vibration analysis method. The photographic method does not allow for to detection of damage hidden inside the body of the turbine blade. The main disadvantage of most non-destructive testing methods is that they are not applicable in non-stationary conditions when rotating turbine blades.

Today, one of the approaches to assessing the destruction of a turbine blade is to consider the blade as a rod experiencing the action of tensile centrifugal and bending gas forces. Considering the problem in such a formulation can lead to a number of errors, however, it can significantly simplify its solution. It should also be noted that the development of a model of blade damage during long-term operation was carried out in [25], and a mathematical model for diagnosing defects in turbine elements was considered in [26].

Thus, according to the analysis of literature sources, it can be concluded that most of the research is aimed at developing models of crack propagation in turbine blades and the use of methods of non-destructive testing of turbine blades in stationary conditions. Scientific works on the detection of damage to turbine blades in engine operating conditions are few.

In studies, [27], [28], [29], [30], a system for diagnosing cracks in turbine blades when the engine is running has been proposed. According to this system, capsules are placed in the body of the turbine blade. Inside the capsule is filled with a substance that exhibits ionizing properties at high temperatures. Capsules are thin-walled shells and are under internal pressure. According to the idea in [27], [28], [29], [30], the crack spreads in the turbine blade and reaches the capsule at some point in time. When a crack opens near the capsule, the capsule breaks due to a pressure difference. The ionizing substance is ejected into the flow part of the turbine, where it is then recorded. This system can be effective in the diagnosis of damage. However, the issues of assessing the uneven pressure inside the capsule, and the optimal geometry of the capsule are unresolved.

Comparing existing models and approaches to crack detection, it should be noted that they are designed to detect cracks in stationary conditions and cannot be applied to monitoring turbine blades on a running engine. In [27], [28], [29], [30], an approach to the diagnosis of damage to turbine blades is proposed, but the calculation of the internal pressure in the capsule is not proposed and the action of centrifugal tensile forces is not taken into account.

## II. Problem statement

Consideration of the problem will be carried out with the introduction of assumptions and simplifications:

1. The cross-section of the turbine blades is constant
2. Working turbine blades are exposed to centrifugal tensile forces and bending gas forces. Bending forces can be neglected since they are an order of magnitude smaller than centrifugal forces.
3. Separation cracks are the most dangerous for turbine blades and develop perpendicular to the blade profile. Therefore, the crack opening will be considered in a cross-section in a perpendicular direction.
4. The shell element of the capsule operates on shear in the crack opening area.

Consider a working turbine blade, which is under the action of centrifugal tensile forces due to rotation at high speeds. Centrifugal forces will affect the gas mixture and the pressure distribution inside the capsule in height due to the rotation of the blade with capsules (Fig. 1). An accurate assessment of the pressure inside the capsule is very important because the capsule will not rupture if there is insufficient pressure at the time of crack opening, which may reduce the likelihood of its diagnosis. Accordingly, there is a need to assess the effect of rotation on the pressure distribution inside the capsule. Another important issue is related to the geometry of the capsule: the dimensions of the capsule shell must be comparable to the dimensions of the turbine blade, and the diameter, and thickness of the capsule cannot be arbitrary in size. In this regard, the inner diameter of the capsule needs to be minimized.

Thus, we define the objectives of the study:

1. Development of a model of pressure changes along the capsule height under the action of centrifugal forces.
2. Development of a model for calculating the minimum internal diameter of the capsule.
3. Estimation of the pressure change inside the capsule in the radial direction for different speeds of rotation of the turbine blade and calculation of the minimum diameter of the capsule for the specified characteristics of the turbine blade

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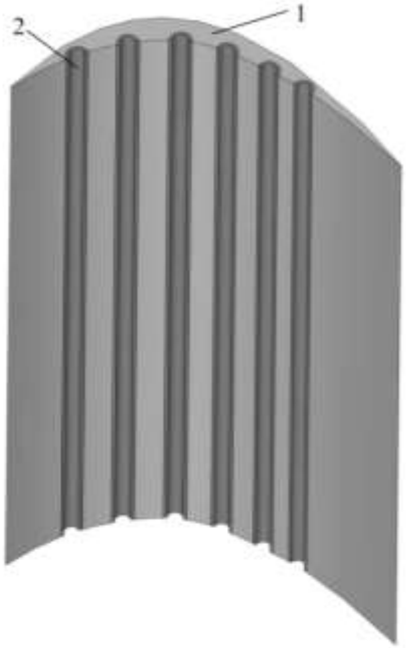


Figure 1. Turbine blade in section: 1 – turbine blade body, 2 – capsule

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## III. RESEARCH METHODOLOGY

The pressure of the gas mixture p is a function of the radial coordinate $p = p(r)$ due to the rotation of the blade. Consider the action of the centrifugal force $F$ on the isolated element of the gas mixture $dV = Sdr$:

$$dF = \omega^2 r dm, \tag{1}$$

where $dr$ is the thickness of the gas mixture layer, $\omega$ is the circumferential velocity, $r$ is the distance from the axis of rotation to the selected element, $dm = pdV$ is the mass of the studied element of the gas mixture, $\rho$ is the density of the gas mixture.

Note that $dF = Sdp$. Let's express the pressure change on the faces of the selected element according to (1):

$$dp = \omega^2 \rho r dr. \tag{2}$$

The gas mixture satisfies the Mendeleev-Clapeyron equation since the equilibrium process is considered in the work:

$$pM = \rho RT, \tag{3}$$

where $M$ is the molar mass of the gas mixture, $R$ is the universal gas constant, $T$ is the temperature.

Using (3), we exclude the density from the relation (2), then

$$\frac{dp}{p} = \frac{\omega^2 M}{RT} r dr. \tag{4}$$

Integrating the relation (4), we obtain:

$$p(r) = C\exp\left(\frac{\omega^2 M}{2RT} r^2\right), \tag{5}$$

where $C$ is the integration constant.

To find the unknown integration constant, we will use the condition for the mixture mass:

$$m = \int_{r_1}^{r_2} \rho S dr, \tag{6}$$

where $r_1$ is the distance from the axis of rotation to the root section, $r_2$ is the distance from the axis to the peripheral section, $m$ is mixture mass.

Then, taking into account (6), we define the constant $C$:

$$C = mRT\left[SM\int_{r_1}^{r_2} \exp\left(\frac{\omega^2 M}{2RT} r^2\right) dr\right]^{-1}. \tag{7}$$

The change in pressure inside the capsule in height is determined according to (5), (7):

$$p(r) = \frac{\exp\left(\frac{\omega^2 M}{2RT} r^2\right) mRT}{SM\int_{r_1}^{r_2} \exp\left(\frac{\omega^2 M}{2RT} r^2\right) dr}. \tag{8}$$

In the second stage of the study, we will consider the calculation of the minimum diameter of the capsule. Let's select a small element of a thin-walled capsule in the crack opening area. Consider the equilibrium condition of the element in shear:

$$p\theta\alpha\frac{D}{2} = \tau\left[\alpha\left(\frac{D}{2} + \delta\right)^2 - \alpha\left(\frac{D}{2}\right)^2\right], \tag{9}$$

where $\theta$ is the width of the crack opening, $D$ is the inner diameter of the capsule, $\alpha$ is the sector angle of the selected element, $\delta$ is the thickness of the capsule wall, $\tau$ is the shear stress.

The limiting state of the capsule element is determined based on the III theory of strength: $\tau = \sigma_s/2$,where $\sigma_s$ is the ultimate strength of the material. We express the inner diameter of the capsule from (9). Then the problem of minimizing the capsule diameter takes the form:

$$D = \frac{\sigma_s \delta^2}{p\theta - \sigma_s \delta} \to \text{min}. \tag{10}$$

To find the smallest value of the diameter D, we require the following condition: $D > 0$, then:

$$0 < \delta < \frac{p\theta}{\sigma_s}. \tag{11}$$

We investigate the function (10) for an extremum in two variables $\delta, p$. Let us differentiate the relation (10):

$$\frac{\partial D}{\partial \delta} = \frac{p\theta^2}{(\sigma_s\delta - p\theta)^2} - 1, \qquad \frac{\partial D}{\partial p} = -\frac{\sigma_s\theta^2}{(\sigma_s\delta - p\theta)^2}. \tag{12}$$

The solution of the system of equations (12): $\delta = 0, p = 0$ does not satisfy the condition (11).

We investigate the function (10) for the extremum of one variable $\delta$ for a fixed variable $p$. Stationary points:

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$$\delta_1 = 0, \delta_2 = \frac{2p\theta}{\sigma_s}$$

do not satisfy the condition (11).

Thus, the smallest diameter is reached at the boundary of the domain of acceptable solutions. The internal pressure in the capsule according to (8) varies according to a nonlinear law, increasing in the radial direction. Accordingly, if we consider capsules of variable cross-section, then the minimum required diameter of the capsule will decrease in the height of the blade because, in the peripheral cross-section, the pressure will be maximum. Substitute (8) in relation (10), then:

$$D \geq D_{\min} = \sigma_s \delta^2 SM \int_{r_1}^{r_2} \exp\left(\frac{\omega^2 M}{2RT} r^2\right) dr \Big/ \left(\exp\left(\frac{\omega^2 M}{2RT} r^2\right) mRT - \sigma_s \delta SM \int_{r_1}^{r_2} \exp\left(\frac{\omega^2 M}{2RT} r^2\right) dr\right). \quad (13)$$

If we consider capsules of constant cross-section, then the assessment of the minimum required diameter of the capsule should be carried out from the condition of the necessary pressure to ensure the rupture of the capsule. Then the minimum diameter of the capsule of a constant cross-section is determined from the ratio:

$$\min D = D|_{p=p|_{r=r_2}} = \text{const}.$$

## IV. CALCULATION RESULTS

We will calculate the uneven pressure in the capsule and the minimum required diameter of the capsules for different speeds of rotation of the rotor of a gas turbine with linear rotation frequencies: 10000 rpm, 12000 rpm, …, 20000 rpm. The $KNO_3$ solution, which exhibits the best ionizing properties at elevated temperatures, will be used as a gas mixture inside the capsule: $M = 0.101$ kg/mol, $m = 9.03 \cdot 10^{-3}$ kg, [28]. The turbine blade is under thermal effect: $T = 773\ K$. Turbine blade geometry: $r_1 = 0.15$ m, $r_2 = 0.25$ m. Specifications of aluminum capsule: $\sigma_s = 100$ MPa, $\delta = 10^{-3}$ m. Crack opening width: $\theta = 10^{-3}$ m.

The results of changing the pressure inside the capsule along the height for different rotation speeds are shown in Fig.2.

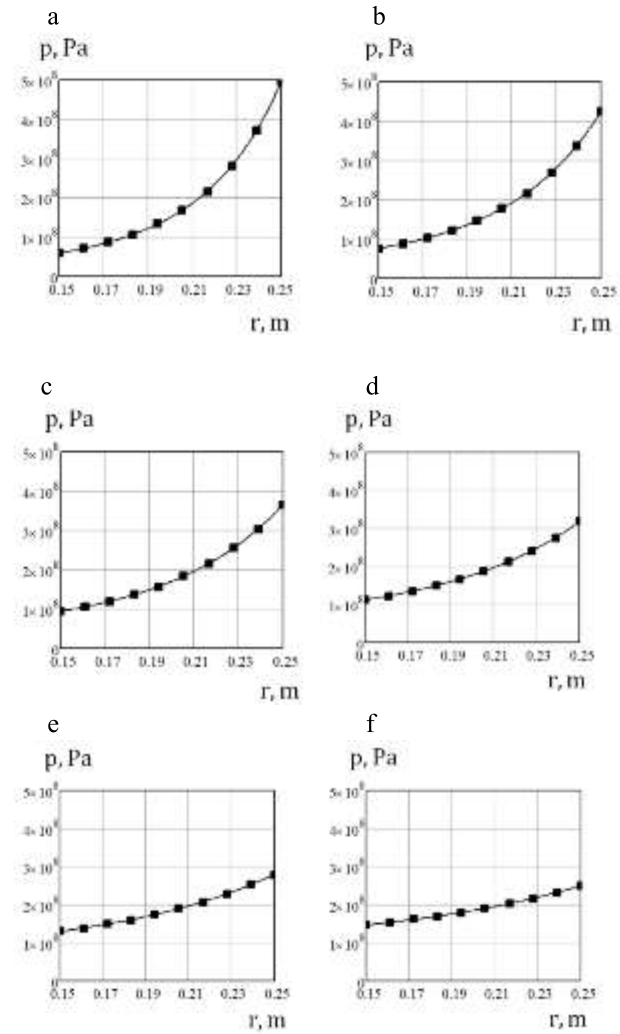


Figure 2. Pressure change in the capsule in the radial direction for the rotation frequencies of the turbine blade: a - 20000 rpm, b - 18000 rpm, c - 16000 rpm, d - 14000 rpm, e - 12000 rpm, f - 10000 rpm

In the absence of centrifugal forces, when the blade is in stationary conditions, the pressure inside the capsule is equally distributed along the height and reaches the value $p|_{\omega=0} = 1.9 \cdot 10^8$ Pa.

The results of changing the minimum diameter of the capsule for different turbine rotation speeds according to (13) are shown in Fig.3.

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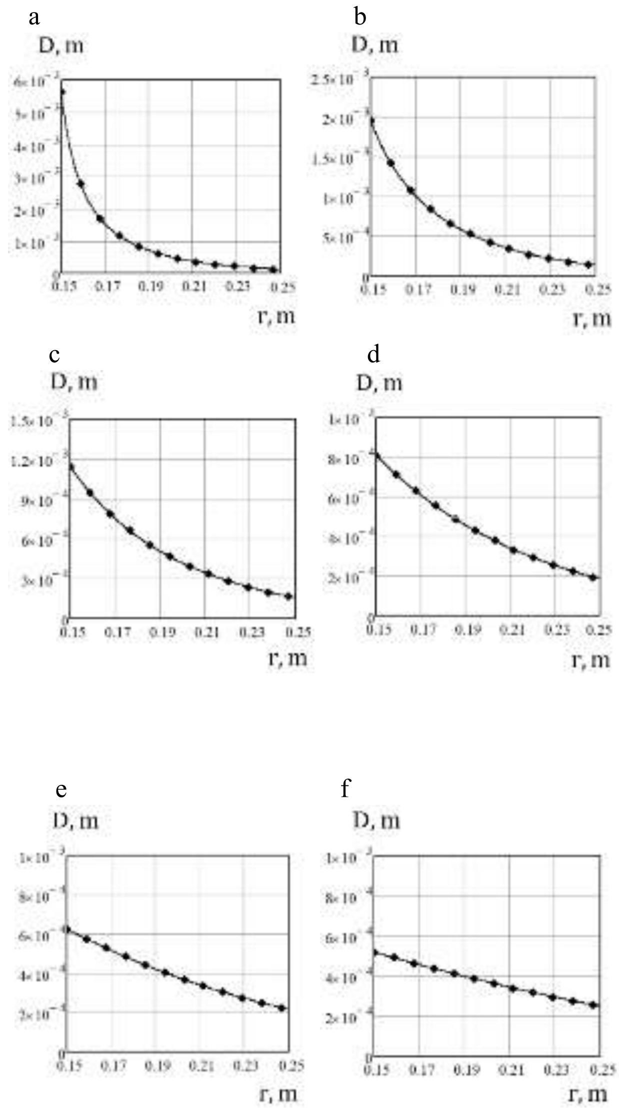


Figure 3. Change of the minimum diameter of the capsule in the radial direction for the rotational speeds of the turbine blade: a - 20000 rpm, b - 18000 rpm, c - 16000 rpm, d - 14000 rpm, e - 12000 rpm, f - 10000 rpm

## V. DISCUSSION OF THE RESULTS

According to the results in Fig. 2, the pressure inside the capsule is distributed unevenly due to the action of centrifugal forces. The pressure varies exponentially in the radial direction. In the area of the root section, the effect of the tensile forces is the smallest, respectively, at a low rotational speed, the pressure is closest to the pressure in the capsule under stationary conditions $p|_{\omega=0}$. As the rotation speed increases, the pressure in the peripheral section of the capsule increases.

The minimum diameter of the capsules varies in height non-linearly according to Fig. 3. The greatest nonlinearity of the dependence of the smallest capsule diameter on the radial coordinate is manifested at a high turbine rotation speed – 20000 rpm. The area above the curves shown in Fig. 3 satisfies the required values for the diameter of the capsules, at which the capsule rupture will be ensured. The minimum values of the diameter of the capsules satisfy the values lying on the curves of Fig.3. The largest value of the capsule diameter is reached in the area of the root section of the turbine blade.

It is important to note that if the action of centrifugal forces is not taken into account, then the error in calculating the pressure in the root section area will be 21% at a frequency of 10000 rpm and 52% at a frequency of 20,000 rpm.

Thus, the results of the study allow us to calculate the exact pressure that will act on the inner surface of the capsule when the turbine blade rotates due to the action of centrifugal forces. Knowing the exact pressure, it is possible to determine the minimum diameter of the capsules at which the capsule will rupture in the event of a crack opening.

## VI. CONCLUSION

The proposed model for calculating the minimum internal diameter of the capsule shell for a system for detecting damage in the blades of a gas turbine engine under non-stationary conditions was developed in the study. The effect of centrifugal forces during the rotation of the turbine blade and the uneven distribution of pressure in the capsules is taken into account in the constructed model. The constructed dependencies allow us to link the characteristics of the damage diagnosis system: uneven pressure inside the capsule, the width of the crack opening near the capsule, and the diameter and thickness of the capsule wall. The use of the proposed model will significantly improve the efficiency of the system for diagnosing cracks in turbine blades under non-stationary conditions due to a more accurate calculation of the loading of the capsule and calculation of its geometry.

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**Contribution of individual authors to the creation of a scientific article (ghostwriting policy)**

-Ivan K. Andrianov carried out the formulation of the problem research and modelling.

-Elena K. Chepurnova carried out a series of calculations of the pressure and diameter of the capsules of the system.

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**Conflict of Interest**

The authors have no conflict of interest to declare that is relevant to the content of this article.