Identification of Parallel-Cascade Wiener System using Tensor Decomposition of an associated Volterra kernel

Zouhour Ben Ahmed, Nabil Derbel
LabCEM, Sfax Engineering School,
BP 1173, 3038 Sfax,
Tunisia

Abstract- In this paper, we propose tensor-based methods for identifying nonlinear Parallel-Cascade Wiener (PCW) systems. Parameters of linear subsystems are first estimated using an approach based on the PARAFAC decomposition of the associated pth-order Volterra kernel. This approach consists in applying the Alternating Least Squares (ALS) algorithm. Then the coefficients of nonlinear subsystems approximated as polynomials are estimated by mean the least square sense from the reconstructed output of the linear subsystems. The proposed parameter estimation method and its performance are illustrated by means of simulation results.

Keywords- Parallel-Cascade Wiener models, identification, PARAFAC decomposition, tensor, Volterra kernels, Alternating Least Squares (ALS).

I. Introduction

Many nonlinear models can be represented by a cascade of linear dynamic subsystems with memoryless (static) nonlinearities, also called block-structured nonlinear models. These types of models have been extensively studied by many authors during the last two decades (see [5] for an extensive bibliography). They play an important role in many fields of application because of their low complexity that imply a low computational cost for system identification. Three types of block-structured nonlinear models are commonly used: Wiener model (a linear dynamic subsystem followed by a nonlinear static one), Hammerstein model (the dual of the Wiener model obtained by reversing the order of the linear dynamic block and the static nonlinearity), and Wiener-Hammerstein (constituted of a nonlinear static subsystem in sandwich between two linear dynamic subsystems). A possible generalization of this class of models gives rise to the parallel-cascade models which consists of combining these simple models in parallel.

This paper is concerned with the study of Parallel cascade Wiener (PCW) type nonlinear structures. This models connect different Wiener systems excited by the same input signal in parallel. They have been successfully employed in various areas, including biological systems [6] and microwave power amplifiers [12].

Several methods have been proposed for identifying such PCW models. Iterative algorithms have been developed for PCW systems [9, 15]. This algorithms are based ,respectively, on the use of one-dimensional slices of input/residual cross-correlation functions and second-order input/residual cross-correlation matrix. Such an approaches does not guarantee a unique representation. So, a new approach has been proposed in [7] for estimating a PCW model based on a joint diagonalization of the associated third-order Volterra kernel. Some works have also proposed an estimation method that combines the knowledge obtained by estimating the best linear approximation of a nonlinear system with a dimension reduction method to estimate the linear time-invariant blocks present in the model [10].

Large numbers of classical data processing techniques depend on the representation of vector and matrix forms, where the vectorization or matricization is much used on multidimensional data. However, important underlying structure information can be lost during processing. Over the last years, tensor models have been well provided as a natural idea for representing systems and data that involve multiple dimensions. They have been studied in a various number of areas, such as signal processing [2, 11], computer vision [14], numerical analysis [1], and more.

In this paper, we propose tensor-based approach for identifying PCW systems that are nonlinear with respect to their parameters. This approach are based on the PARAFAC decomposition of pth-order Volterra kernel associated with the system to be identified. This Volterra kernel that is treated as a symmetric tensor, can be estimated by means of i.i.d. inputs as shown in [8]. By considering the pth-order Volterra kernel associated with a PCW system as a tensor, we show that the linear subsystems can be estimated using Alternating Least Squares

(ALS) algorithm. In a second step, the coefficients of the nonlinear subsystem modeled as a polynomial are estimated by means of the RLS algorithm.

The rest of this paper is organized as follows. Section 2 describes the nonlinear PCW system and gives the expression of the Volterra kernels associated. Tensors and PARAFAC Decomposition are briefly reviewed in section 3. In Section 4, we present tensor-based approaches for identifying PCW systems. The proposed identification method is illustrated by means of some simulation results in Section 5, before concluding the paper in Section 6.

Notations : Scalars, vectors, matrices and high-order tensors are written as lower-case $(a, b, \cdots)$, bold lower-case $(\mathbf{a}, \mathbf{b}, \cdots)$, bold upper-case $(\mathbf{A}, \mathbf{B}, \cdots)$ and blackboard $(\mathbb{A}, \mathbb{B}, \cdots)$ letters, respectively. $\mathbf{A}^T$ and $\mathbf{A}^+$ denote transpose and Moore-Penrose pseudo-inverse of $\mathbf{A}$, respectively. The vector $\mathbf{A}_{i.}$ (resp. $\mathbf{A}_{.j}$) represent the i^{th} row (resp. j^{th} column) of $\mathbf{A}$. Scalars $a_i, a_{i,j}$ and $a_{i_1,\cdots,i_N}$ indicate, respectively, the i^{th} element of $\mathbf{a}$, the $(i,j)^{th}$ element of $\mathbf{A}$ and the $(i_1,\cdots,i_N)^{th}$ element of $\mathbb{A}$. $\mathbf{e}_k^{(K)}$ denotes the kth unit vector of the Euclidean basis in $\mathcal{R}^K$ and $\|.\|$ is the Euclidean norm. $\mathbf{1}_N$ and $\mathbf{I}_N$ denote respectively the all ones vector of dimension N and the identity matrix of order N. The operator $vec(.)$ forms a vector by stacking columns of its matrix argument, while $unvec_{I\times J}(.)$ is its inverse operator that forms a $I \times J$ matrix from its vector argument of dimensions $IJ \times 1$. The operator $diag(.)$ forms a diagonal matrix from its vector argument. $\circ$, $\otimes$ and $\diamond$ denote, respectively, The outer, Kronecker, and Khatri-Rao products. For matrices $\mathbf{A}$, $\mathbf{B}$, $\mathbf{C}$ and vector $\mathbf{v}$, we have :

$$vec(\mathbf{A}diag(\mathbf{v})\mathbf{B}) = (\mathbf{B}^T \diamond \mathbf{A})\mathbf{v} \tag{1}$$

$$vec(\mathbf{A}\mathbf{C}\mathbf{B}^T) = (\mathbf{B} \otimes \mathbf{A})vec(\mathbf{C}) \tag{2}$$

II. PCW models and its associated Volterra kernels

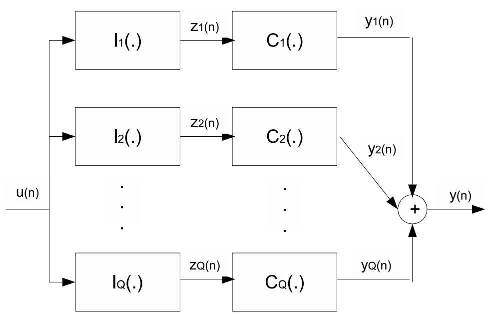

Fig. 1: Parallel-cascade Wiener model

Let us consider the nonlinear PCW model illustrated in Figure 1. The output $y(.)$ of this PCW system is reached from the sum of the outputs $y_q(.)$ of the parallel paths: $y(n) = \sum_{q=1}^{Q} y_q(n)$. Each path is a Wiener nonlinear system composed of a linear filter subsystem with impulse response $l_q(.)$ and memory M_q and a memoryless nonlinear subsystem $C_q(.)$ approximated by means of a finite degree polynomial of degree P_q, with coefficients $c_{p,q}$. So, the output of a PCW model is described by means of the following equations:

$$\begin{aligned} y(n) &= \sum_{q=1}^{Q}\sum_{p=1}^{P_q} c_{p,q} z_q^p(n) \\ &= \sum_{q=1}^{Q}\sum_{p=1}^{P_q} c_{p,q} \left(\sum_{i=0}^{M_q-1} l_q(i)u(n-i)\right)^p \end{aligned} \tag{3}$$

We can note that the PCW system representation is not linear with respect to its parameters. Instead of estimating the parameters of each block by using a nonlinear optimization method, it is more suitable to estimate the equivalent Volterra model, which is linear in its parameters. Indeed, the output of the p^{th} path can be written as:

$$y(n) = \sum_{p=1}^{P_q} \sum_{i_1,\cdots,i_p=0}^{M_q-1} h_{p,q}(i_1,\cdots,i_p) \prod_{k=1}^{p} u(n-i_k) \tag{4}$$

where $h_{p,q}(i_1,\cdots,i_p)$ denotes the p^{th} order Volterra kernel associated with the p^{th} path, given by:

$$h_{p,q}(i_1,\cdots,i_p) = c_{p,q}\prod_{k=1}^{p} l_q(i_k), \;\; i_k = 0,\cdots,M_q-1 \tag{5}$$

Then, the input-output relationship of the PCW system is given by:

$$y(n) = \sum_{p=1}^{P} \sum_{i_1,\cdots,i_p=0}^{M-1} h_p(i_1,\cdots,i_p) \prod_{k=1}^{p} u(n-i_k) \tag{6}$$

where

$$h_p(i_1,\cdots,i_p) = \sum_{q=1}^{Q} c_{p,q}\prod_{k=1}^{p} l_q(i_k), \;\; i_k = 0,\cdots,M-1$$

$P = \max_q P_q$, $M = \max_q M_q$.

III. Tensors and PARAFAC Decomposition

A tensor of n modes (or n-way) is a structure indexed by n variables. For example, a matrix is a two-way tensor. Let $\mathbb{H}$ be a tensor of order N with dimensions $I_1 \times I_2 \times \cdots \times I_N$ and entries $h_{i_1,i_2,\cdots,i_n}$ with $i_j = 1,2,\cdots,I_j$ and $j = 1,2,\cdots,N$.
PARAFAC Decomposition: PARAFAC decomposition approximates the original tensor $\mathbb{H}$ with a model which can be expressed as a sum of R rank-one tensors:

$$\mathbb{H} = \sum_{r=1}^{R} \mathbf{A}_{.r}^{(1)} \circ \mathbf{A}_{.r}^{(2)} \circ \cdots \circ \mathbf{A}_{.r}^{(N)} \tag{7}$$

where $\mathbf{A}_{.r}^{(n)}$ is the rth column of the factor matrix $\mathbf{A}^{(n)} \in \mathbb{R}^{I_n\times R}$, $n = 1,\cdots,N$. This decomposition can be written in the following scalar form:

$$h_{i_1 i_2 \cdots i_N} = \sum_{r=1}^{R}\prod_{n=1}^{N} a_{i_n r}^{(n)}, \; i_n = 1,\cdots,I_n. \tag{8}$$

$a_{i_n r}^{(n)}$ being the entries of the factor matrix $\mathbf{A}^{(n)}$.
Essentially, for obtaining the PARAFAC decomposition, we have to solve the following optimization problem:

$$\min_{\mathbf{A}^{(n)}} \|\mathbb{H} - \sum_{r=1}^{R} \mathbf{A}_{.r}^{(1)} \circ \mathbf{A}_{.r}^{(2)} \circ \cdots \circ \mathbf{A}_{.r}^{(N)}\|_F^2 \tag{9}$$

The most popular formulation for fitting the PARAFAC decomposition is the Alternating Least Squares (ALS). The computational complexity of this Algorithm for a $I_1 \times I_2 \times \cdots \times I_N$ tensor and for R components is $O(I_1 I_2 \cdots I_N R)$ per iteration.

IV. Tensor-based method for PCW systems identification

A. Estimation of Volterra kernels associated with PCW models

There are several algorithms for estimating the Volterra kernel. For cubic systems and pth-order, closed-form expression are derived in [13] and [8], respectively, using independent and identically distributed (i.i.d.) input signals. For an arbitrary degree of non-linearity, such expressions have also been derived for circular inputs [4], and in the case of a phase shift keying (PSK)-modulated input as considered in [16].
In this paper, the Volterra kernel associated with the PCW models can be estimated by means a closed-form expression like that derived in [8] using i.i.d. inputs. We can note that the highest order kernels are independent from the lower order ones, the contrary being false. As a consequence, the estimation precision of the third-order kernel depends on that of the higher-order ones. Whereas, the proposed identification methods are based on the use of the fifth-order kernel. In this case we limit ourselves to estimating only the fifth-order kernel instead of estimating several kernels ($p = 3, 4, 5$) as suggested in [3].
The closed-form expressions of Volterra kernel estimates have been derived under the following assumptions:

- The input/output signals are ergodic and stationary at least up to the sixth-order.
- The additive noise is zero-mean and independent of the input signal.
- The input signal is real-valued, zero-mean, i.i.d. with a symmetrical probability distribution function.

For higher-order models (> 5), determining similar expressions is a tough task that was not yet considered in the literature.

B. Estimation of linear subsystem

Let define the matrix $\mathbf{L} \in \mathcal{R}^{M \times Q}$ containing the impulse response of the Q linear subsystems, i.e., the matrix with $\mathbf{L}_{.q} = (l_q(0), \cdots, l_q(M-1))$, $q = 1, \cdots, Q$, and we formulate the following assumptions [7]:

- A1) $\mathbf{L}$ is a full column rank matrix, which denotes $M \geq Q$,
- A2) $l_q(0) = 1,\ q = 1, \cdots, Q$,
- A3) $c_{p,q} \neq 0,\ \forall q$.

For a PCW model, the associated p^{th}-order Volterra kernel can be seen as a p^{th}-order symmetric tensor denoted by $\mathbb{H}_p \in \mathcal{R}^{M \times M \times \cdots \times M}$ of rank $R = Q$. It can always be decomposed as:

$$\mathbb{H}_p = \sum_{r=1}^{R} \mathbf{A}_{.r}^{(1)} \circ \mathbf{A}_{.r}^{(2)} \circ \cdots \circ \mathbf{A}_{.r}^{(p)}, \tag{10}$$

with $\mathbf{A}^{(n)} \in \mathbb{R}^{M \times Q},\ n = 1, \cdots, N$ are the matrix factors of the PARAFAC decomposition of the tensor $\mathbb{H}_p$. We get then the following expressions for the PARAFAC factors and the mode-P unfolded matrix representation of $\mathbb{H}_P$, denoted by $\mathbf{H}_p \in \mathcal{R}^{M^{p-1} \times M}$:

$$\mathbf{H}_p = \left(\mathbf{A}^{(1)} \diamond \mathbf{A}^{(2)} \diamond \cdots \diamond \mathbf{A}^{(p-1)}\right) \mathbf{A}^{(p)^T} \tag{11}$$

$$= (\mathbf{L} \diamond \mathbf{L} \diamond \cdots \diamond \mathbf{L})\, diag(\mathbf{c}_p) \mathbf{L}^T \tag{12}$$

with $\mathbf{A}^{(1)} = \mathbf{A}^{(2)} = \cdots = \mathbf{A}^{(p-1)} = \mathbf{L}$, $\mathbf{A}^{(p)} = \mathbf{L} diag(\mathbf{c}_p)$ where $\mathbf{c}_p = (c_{p,1}, \cdots, c_{p,Q})^T$.
Using equation (1) gives :

$$vec(\mathbf{H}_p) = (\mathbf{L} \diamond \mathbf{L} \diamond \mathbf{L} \diamond \mathbf{L} \diamond \mathbf{L}) \mathbf{c}_p \tag{13}$$

The LS update of $\mathbf{c}_p$ is given by :

$$\hat{\mathbf{c}}_p = (\mathbf{L} \diamond \mathbf{L} \diamond \mathbf{L} \diamond \mathbf{L} \diamond \mathbf{L})^+ vec(\mathbf{H}_p) \tag{14}$$

Another matrix unfolding of the kernel tensor is given by:

$$\mathbf{H}_1 = (\mathbf{A}^{(2)} \diamond \cdots \diamond \mathbf{A}^{(p)}) \mathbf{A}^{(1)^T} \tag{15}$$

So, we have :

$$\mathbf{H}_1^T = \mathbf{I}_M \mathbf{A}^{(1)} (\mathbf{A}^{(2)} \diamond \cdots \diamond \mathbf{A}^{(p)})^T \tag{16}$$

Using equation (2), we obtain :

$$vec(\mathbf{H}_1^T) = ((\mathbf{A}^{(2)} \diamond \cdots \diamond \mathbf{A}^{(p)}) \otimes \mathbf{I}_M) vec(\mathbf{A}^{(1)})$$
$$= (\mathbf{L} \diamond \cdots \diamond \mathbf{L} \diamond (\mathbf{L} diag(\mathbf{c}_p)) \otimes \mathbf{I}_M) vec(\mathbf{L}) \tag{17}$$

Defining :

$$\mathbf{Q} = (\mathbf{L} \diamond \cdots \diamond \mathbf{L} \diamond (\mathbf{L} diag(\mathbf{c}_p)) \otimes \mathbf{I}_M \tag{18}$$

The LS update of $\mathbf{L}$ is then given by :

$$vec(\hat{\mathbf{L}}) = \mathbf{Q}^+ vec(\mathbf{H}_1^T) \tag{19}$$

The ALS algorithm for identifying the PCW system is summarized in Table 1.

C. Estimation of nonlinear subsystem

After estimating the impulse response coefficients $l_q(.)$, the PCWS model can be presented as:

$$\hat{y}(n) = \sum_{q=1}^{Q} \sum_{p=1}^{P} c_{p,q} \hat{z}_q^p(n) = \mathbf{z}^T(n) \mathbf{c} \tag{20}$$

Table 1: ALS algorithm

Given the unfolded matrix $\mathbf{H}_1$ and $\mathbf{H}_p$ of the tensor $\mathbb{H}_p$ corresponding to the pth-order Volterra kernel associated with the PCW system,

1) $k = 0$: initialize $\hat{\mathbf{L}}_0$ with random values and $\hat{\mathbf{L}}_{0_{1.}} = \mathbf{1}$,
2) $k = k + 1$,
3) Compute $\hat{\mathbf{c}_p}^k = (\hat{\mathbf{L}}_{k-1} \lozenge \cdots \lozenge \hat{\mathbf{L}}_{k-1})^+ vec(\mathbf{H_p})$,
4) Deduce $\hat{\mathbf{Q}}^k = (\hat{\mathbf{L}}_{k-1} \lozenge \cdots \lozenge \hat{\mathbf{L}}_{k-1} \lozenge (\hat{\mathbf{L}}_{k-1} diag(\hat{\mathbf{c}_p}^k))) \otimes \mathbf{I}_M$,
5) Compute $vec(\hat{\mathbf{L}}_k) = \mathbf{Q}^{k^+} vec(\mathbf{H}_1^T)$, then construct the matrix $\hat{\mathbf{L}}_k$,
6) Return to step 2) until a stop criterion is reached,
7) Normalize $\hat{\mathbf{L}}$ by the equation $\hat{\mathbf{L}} = \hat{\mathbf{L}} diag(\hat{\mathbf{L}}_{1.})^{-1}$.

where $\hat{z}_q(n)$ denotes the reconstructed output of the q^{th} FIR subsystem, i.e., $\hat{z}_q(n) = \sum_{i=0}^{M-1} \hat{l}_q(i) u(n-i)$. By concatenating the model outputs for $n = 1, \cdots, N$, we get:

$$\hat{\mathbf{y}} = \mathbf{Z}\mathbf{c} \tag{21}$$

with $\mathbf{Z} = (\hat{\mathbf{z}}(1) \ \cdots \ \hat{\mathbf{z}}(N))^T$, $\hat{\mathbf{y}} = (\hat{y}(1) \ \cdots \ \hat{y}(N))^T$ and $\mathbf{c} = (c_{1,1} \ c_{2,1} \ \cdots \ c_{P,1} \ \cdots \ c_{1,Q} \ \cdots \ c_{P,Q})^T$.
The LS estimate of the polynomial coefficients of the nonlinear subsystem is given by :

$$\hat{\mathbf{c}} = \mathbf{Z}^+ \mathbf{y}. \tag{22}$$

D. Summary

The proposed identification method is composed of the three following steps :

1) Estimate the Pth-order Volterra kernel associated with the PCW system to be identified using input-output measurements using the method proposed in [8].

2) Estimate the linear subsystems parameters by means of an ALS algorithm using the PARAFAC decomposition of the estimated Volterra kernel in step (1) (table 1).

3) Estimate the coefficients of the nonlinear subsystems by applying the LS solution 22.

V. Simulations Results

Simulation results were obtained with $Mn = 100$ different PCW models, with memory $M = 3$ and nonlinearity degree $P = 5$. Moreover, $B = 10$ different additive, zero-mean, white Gaussian noise sequences were added to each model output with fixed SNR (Signal-to-Noise Ratio).
Performances are evaluated in terms of Normalized Mean Square Error (NMSE) on the output signal (NMSE$_s$), on the estimated parameters of the linear subsystem (NMSE$_l$) and of the nonlinear subsystem (NMSE$_c$).

$$\text{NMSE}_l = 10\log\left(\frac{1}{S_c}\sum_{m=1}^{Mn}\sum_{b=1}^{B}\frac{\|\hat{\mathbf{L}}_{m,b}-\mathbf{L}_m\|_2^2}{\|\mathbf{L}_m\|_2^2}\right),$$

$$\text{NMSE}_c = 10\log\left(\frac{1}{S_c}\sum_{m=1}^{Mn}\sum_{b=1}^{B}\frac{\|\hat{\mathbf{c}}_{m,b}-\mathbf{c}_m\|_2^2}{\|\mathbf{c}_m\|_2^2}\right),$$

$$\text{NMSE}_{h_p} = 10\log\left(\frac{1}{S_c}\sum_{m=1}^{Mn}\sum_{b=1}^{B}\frac{\|\hat{\mathbb{H}}_{m,b}-\mathbb{H}_m\|_2^2}{\|\mathbb{H}_m\|_2^2}\right),$$

$$\text{NMSE}_s = 10\log\left(\frac{1}{S_c}\sum_{m=1}^{Mn}\sum_{b=1}^{B}\frac{\|\hat{\mathbf{s}}_{m,b}-\mathbf{s}_m\|_2^2}{\|\mathbf{s}_m\|_2^2}\right)$$

where $\mathbf{s}_m$ denotes the output vector associated with the m^{th} simulated model, whereas $\hat{\mathbf{s}}_{m,b}$ denotes the vector of reconstructed output using the estimated parameters $\hat{\mathbf{L}}_{m,b}$ and $\hat{\mathbf{c}}_{m,b}$, for the m^{th} simulated model and the b^{th} run. $\hat{\mathbb{H}}_{m,b}$ denotes the reconstructed fifth-order Volterra kernel associated with the m^{th} simulated model and b^{th} noise sequence. S_c denotes the number of runs that converged.
Standard deviations of the parameter estimates for the m^{th} simulated model is given by :

$$E_{\mathbf{l}} = \sqrt{\frac{1}{B_c}\sum_{b=1}^{B}\left(\hat{\mathbf{L}}_{m,b}-\hat{\mathbf{L}}_m\right)^2}, \text{ with } \hat{\mathbf{L}}_m = \frac{1}{B_c}\sum_{b=1}^{B}\hat{\mathbf{L}}_{m,b}$$

$$E_{\mathbf{c}} = \sqrt{\frac{1}{B_c}\sum_{b=1}^{B}(\hat{\mathbf{c}}_{m,b}-\hat{\mathbf{c}}_m)^2}, \text{ with } \hat{\mathbf{c}}_m = \frac{1}{B_c}\sum_{b=1}^{B}\hat{\mathbf{c}}_{m,b}$$

where B_c denotes the number of runs that converged for the m^{th} model.
The input signal was an i.i.d. 6-RMS (Random Multilevel Sequence, with six levels) signal with length N ($\{\pm1, \pm2/3, \pm1/3\}$). To guarantee the i.i.d. property, the input sequence was designed according to the method described in [13].

Table 2: NMSE$_l$, NMSE$_c$, NMSE$_{h_5}$ and NMSE$_s$ for different values of SNR ($Q = 3$,$M = 3$, $P = 5$)

	SNR= 0dB	SNR= 10dB	SNR= 40dB
NMSE$_l$ (dB)	-15.84	-20.60	-56.16
NMSE$_c$ (dB)	4.98	-4.62	-38.48
NMSE$_{h_5}$ (dB)	-10.10	-19.23	-52.45
NMSE$_s$ (dB)	-2.78	-10.13	-39.78

Table 2 presents the results obtained with a PCW system having three branches and an input length $N = 21600$. From these simulation results, we can conclude that as expected, the NMSE$_s$, NMSE$_{h_5}$, NMSE$_l$ and NMSE$_c$ decrease when the SNR increases. We can note also that the output NMSE corresponds approximately to the noise level, implying that the simulated PCW model is well estimated.
We consider different configurations by varying the memory value M and the pathway number Q. Figure 2 illustrate the NMSE$_s$ for two different values of Q and

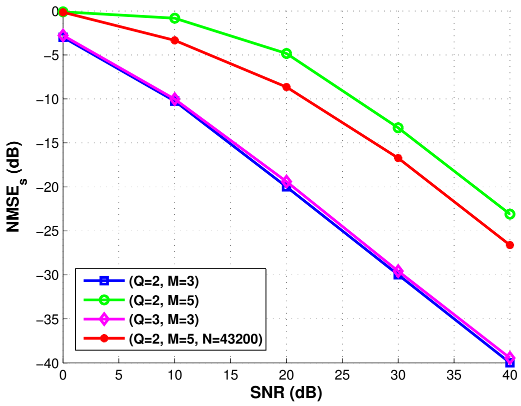

Fig. 2: NMSE_s for different PCW system configurations

two memory. We can notice that the proposed method provides good performances. When the memory or/and the pathway number increases, the performances are degraded. This degradation can be repaid by increasing the data number N, as shown in Figure 2.

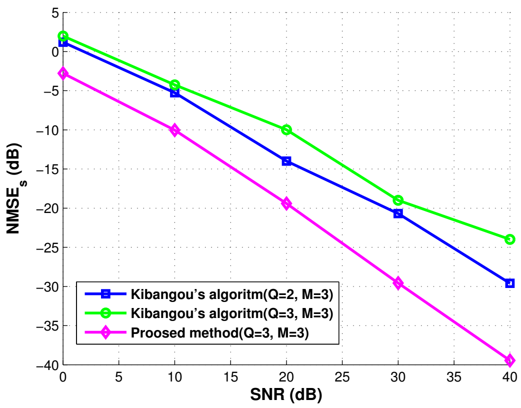

Fig. 3: Comparison with the Kibangou's algorithm [7]

Figure 3 depicts the output NMSE attained with the proposed method and the parallel-cascade identification algorithm in [7]. We can note that, with Kibangou's algorithm, when the number of paths increases give a slight performance improvement. However, the proposed method widely outperforms the Kibangou's algorithm since it takes the fifth-order volterra kernel instead of the third-order. Proposed method allow to reduce the computational cost owing to the estimation of only the fifth-order Volterra kernel without requiring the estimation of the overall Volterra model, and reduce the errors caused by error propagation from calculating several kernels as in [7].

VI. Conclusion

In this paper, we have proposed a tensor-based method for identifying the paths of a parallel-cascade Wiener system. This method is accomplished in three steps. First, the p^{th}-order kernel of the associated Volterra model is estimated using input-output measurements. Second, the linear subsystems are estimated by applying an alternating least square algorithm to the p^{th}-order Volterra kernel slices associated with the PCW to be identified. Then, the coefficients of the nonlinear subsystems are estimated by means the LS algorithm. The efficiency of the proposed estimation method has been illustrated by means of simulation results.

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Zouhour Ben Ahmed received her Engineering Diploma in Applied Informatics in 2009, Master diploma in Intelligent and communicant Systems in 2010 from the National School of Engineering of Sousse, Tunisia, and (Ph.D) and State Doctorate degrees in Electrical Engineering from the National Engineering School of Sfax, Tunisia in 2015. Her research is centred on development of algorithms for nonlinear systems identification, tensor models/decompositions, and tensor approaches for nonlinear block-structured systems.

Nabil Derbel received his Engineering Diploma from the National School of Engineering of Sfax in 1986, DEA diploma in Automatics from INSA of Toulouse in 1986, PhD degree from the Laboratoire LAAS of Toulouse in 1989, and Doctorat d'Etat degree from the National School of Engineering of Tunis in 1997. He joined the Tunisian University in 1989, where he held different positions involved in research and education. Currently, he is a Professor of Automatic Control at the National School of Engineering of Sfax. His current interests include optimal control, sliding mode control, fuzzy systems, neural networks, etc