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On Asymptotically $\mathcal{I}$-Lacunary statistical Equivalent Sequences of order α

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Abstract—This paper presents the following definition which is a natural combination of the definition for asymptotically equivalent of order α ,where $0<\alpha<1$, $\mathcal{I}$ -statistically limit, and $\mathcal{I}$ -lacunary statistical convergence. Let θ be a lacunary sequence; the two nonnegative sequences $x=(x_k)$ and $y=(y_k)$ are said to be asymptotically $\mathcal{I}$ -lacunary statistical equivalent of order α to multiple L provided that for every $\varepsilon>0$, and $\delta>0$,

$$\{r\in\mathbb{N}:\frac{1}{h_r^{\alpha}}|\{k\in I_r:|\frac{x_k}{y_k}-L|\geq\varepsilon\}|\geq\delta\}\in\mathcal{I},$$

(denoted by $x \overset{S_\theta^L(I)^\alpha}{\sim} y$) and simply asymptotically $\mathcal{I}$ -lacunary statistical equivalent of order α if $L=1$. In addition, we shall also present some inclusion theorems.The study leaves some interesting open problems.

Keywords—Asymptotical equivalent, ideal convergence, $\mathcal{I}$ -statistical convergence, $\mathcal{I}$ -lacunary statistical convergence, statistical convergence of order α .

INTRODUCTION

The concept of statistical convergence was introduce by Fast [4] in 1951.

A sequence (x_k) of real numbers is said to be statistically convergent to L if for arbitrary $\varepsilon>0$,

$$\frac{1}{n}|\{k<n:|x_k-L|\geq\varepsilon\}|=0,$$

where by $k<n$ we mean that $k=0,1,2,...,n$ and the vertical bars indicate the number of elements in the enclosed set. In this case we write $st-lim x=L$ or $x_k\mapsto L(st)$.

The idea of statistical convergence was further extended to $\mathcal{I}$ -convergence in [7] using the notion of ideals of $\mathbb{N}$ with many interesting consequences.

By a lacunary $\theta=(k_r);\ r=0,1,2,...$ where $k_0=0$, we shall mean an increasing sequence of non-negative integers with $k_r-k_{r-1}\to\infty$ as $r\to\infty$. The intervals determined by θ will be denoted by $I_r=(k_{r-1},k_r]$ and $h_r=k_r-k_{r-1}$. The ratio $\frac{k_r}{k_{r-1}}$ will be denoted by q_r .

Moreover, the following concept is due to Fridy and Orhan[6].

A sequence (x_k) of real numbers is said to be lacunary statistically convergent to L (or S_θ -convergent to L) if for any $\varepsilon>0$,

$$\lim_{r\to\infty}\frac{1}{h_r}|\{k\in I_r:|x_k-L|\geq\varepsilon\}|=0$$

where $|A|$ denotes the cardinality of $A\subset\mathbb{N}$.

Recently in ([3] and [11]), we used ideals to introduce the concepts of $\mathcal{I}$ -statistical convergence and $\mathcal{I}$ -lacunary statistical convergence which naturally extend the notions of the above mentioned convergence.

On the other hand, in [1] a different direction was given to the study of statistical convergence where the notion of statistical convergence of order α , $0<\alpha<1$ was introduced by replacing n by n^α in the denominator in the definition of statistical convergence. One can also see [2] for related works.

In 1993 Marouf [9] presented definitions for asymptotically equivalent sequences and asymptotic regular matrices.Also,in 1997, Li [8] presented and studied asymptotic equivalence of sequences and summability. In 2003, Patterson [10] extended these concepts by presenting an asymptotically statistical equivalent analog of these definitions and natural regularity conditions for nonnegative summability matrices.

In present paper, we intend to unify these two approaches and we use asymptotical equivalent to introduce the concept asymptotically $\mathcal{I}$ -statistical equivalent of order α and asymptotically $\mathcal{I}$ -lacunary statistical equivalent of order α . In addition to these definitions, natural inclusion theorems shall also be presented.

Throughout by two sequences $x=(x_k)$ and $y=(y_k)$ we shall mean two sequences of real numbers.

I. MAIN RESULT

The following definitions and notions will be needed in the sequel.

Definition 1.(Marouf, [9]) Two nonnegative sequences $x=(x_k)$ and $y=(y_k)$ are said to be asymptotically equivalent if

$$\lim_k\frac{x_k}{y_k}=1$$

(denoted by $x\sim y$).

Definition 2.(Fridy, [5]) The sequence $x=(x_k)$ has statistic limit L , denoted by $st-\lim s=L$ provided that for every $\varepsilon>0$,

$$\lim_n \frac{1}{n}\{\text{the number of } k \le n : |x_k - L| \ge \varepsilon\} = 0.$$

The next definition is natural combination of definitions 1 and 2.

Definition 3.(Patterson, [10]) Two nonnegative sequences $x=(x_k)$ and $y=(y_k)$ are said to be asymptotically statistical equivalent of multiple L provided that for every $\varepsilon>0$,

$$\lim_n \frac{1}{n}\{\text{the number of } k < n : |\frac{x_k}{y_k} - L| \ge \varepsilon\} = 0$$

(denoted by $x \overset{S_L}{\sim} y$), and simply asymptotically statistical equivalent if $L=1$.

Definition 4. A family $\mathcal{I} \subset 2^{\mathbb{N}}$ is said to be an ideal of $\mathbb{N}$ if the following conditions hold:

(a) $A, B \in \mathcal{I}$ implies $A \cup B \in \mathcal{I}$,

(b) $A \in \mathcal{I}, B \subset A$ implies $B \in \mathcal{I}$,

Definition 5. A non-empty family $F \subset 2^{\mathbb{N}}$ is said to be an filter of $\mathbb{N}$ if the following conditions hold:

(a) $\varphi \notin F$,

(b) $A, B \in F$ implies $A \cap B \in F$,

(c) $A \in F, A \subset B$ implies $B \in F$,

If $\mathcal{I}$ is a proper ideal of $\mathbb{N}$ (*i.e.*, $\mathbb{N} \notin \mathcal{I}$), then the family of sets $F(\mathcal{I}) = \{M \subset \mathbb{N} : \exists A \in \mathcal{I} : M = \mathbb{N} \setminus A\}$ is a filter of $\mathbb{N}$. It is called the filter associated with the ideal.

Definition 6. A proper ideal $\mathcal{I}$ is said to be admissible if $\{n\} \in \mathcal{I}$ for each $n \in \mathbb{N}$.

Throughout $\mathcal{I}$ will stand for a proper admissible ideal of $\mathbb{N}$.

Definition 7.([7]) Let $\mathcal{I} \subset 2^{\mathbb{N}}$ be a proper admissible ideal in $\mathbb{N}$. Then the sequence (x_k) of elements of $\mathbb{R}$ is said to be $\mathcal{I}$-convergent to $L \in \mathbb{R}$ if for each $\varepsilon > 0$ the set $A(\varepsilon) = \{k \in \mathbb{N} : |x_k - L| \ge \varepsilon\} \in \mathcal{I}$.

We now introduce our main definitions.

Definition 8. A sequence $x=(x_k)$ is said to be $\mathcal{I}$-statistically convergent of order α to L or $S(\mathcal{I})^\alpha$-convergent to L, where $0 < \alpha \le 1$, if for each $\varepsilon > 0$ and $\delta > 0$

$$\{n \in \mathbb{N} : \frac{1}{n^\alpha} |\{k \le n : |x_k - L| \ge \varepsilon\}| \ge \delta\} \in \mathcal{I}.$$

In this case we write $x_k \to L(S(\mathcal{I})^\alpha)$. The class of all $\mathcal{I}$-statistically convergent sequences of order α will be denoted by simply $S(\mathcal{I})^\alpha$.

Also the next definition is natural combination of definitions 1 and 8.

Definition 9. The two nonnegative sequences $x=(x_k)$ and $y=(y_k)$ are said to be asymptotically $\mathcal{I}$- statistical equivalent of order α to multiple L, where $0<\alpha\le 1$, provided that for each $\varepsilon>0$ and $\delta>0$

$$\{n \in \mathbb{N} : \frac{1}{n^\alpha} |\{k \le n : |\frac{x_k}{y_k} - L| \ge \varepsilon\}| \ge \delta\} \in \mathcal{I},$$

(denoted by $x \overset{S^L(\mathcal{I})^\alpha}{\sim} y$) and simply asymptotically $\mathcal{I}$-statistical equivalent of order α if $L=1$. Furthermore, let $S^L(\mathcal{I})^\alpha$ denote the set of x and y such that $x \overset{S^L(\mathcal{I})^\alpha}{\sim} y$.

Remark 1. If $\mathcal{I} = \mathcal{I}_{fin} = \{A \subseteq \mathbf{N} : A$ is a finite subset$\}$, asymptotically $\mathcal{I}$- statistical equivalent of order α to multiple L coincides with asymptotically statistical equivalent of order α to multiple L. For an arbitrary ideal $\mathcal{I}$ and for $\alpha=1$ it coincides with asymptotically $\mathcal{I}$- statistical equivalent of multiple L. When $\mathcal{I}=\mathcal{I}_{fin}$ and $\alpha=1$ it becomes only asymptotically statistical equivalent of multiple L, [10].

Definition 10. Let θ be a l acunary sequence. A sequence $x=(x_k)$ is said to be $\mathcal{I}$-lacunary statistically convergent of order α to L or $S_\theta(I)^\alpha$-convergent to L if for any $\varepsilon>0$ and $\delta>0$

$$\{r \in \mathbb{N} : \frac{1}{h_r^\alpha} |\{k \in I_r : |x_k - L| \ge \varepsilon\}| \ge \delta\} \in \mathcal{I}.$$

In this case we write $x_k \to L(S_\theta(\mathcal{I})^\alpha)$. The class of all $\mathcal{I}$-lacunary statistically convergent sequences of order α will be denoted by $S_\theta(\mathcal{I})^\alpha$.

We now have

Definition 11. Let θ be a l acunary sequence; the two nonnegative sequences $x=(x_k)$ and $y=(y_k)$ are said to be asymptotically $\mathcal{I}$-lacunary statistical equivalent of order α to multiple L provided that for any $\varepsilon>0$ and $\delta>0$

$$\{r \in \mathbb{N} : \frac{1}{h_r^\alpha} |\{k \in I_r : |\frac{x_k}{y_k} - L| \ge \varepsilon\}| \ge \delta\} \in \mathcal{I},$$

(denoted by $x \overset{S_\theta^L(I)^\alpha}{\sim} y$) and simply asymptotically $\mathcal{I}$-lacunary statistical equivalent of order α if $L=1$. Furthermore, let $S_\theta^L(\mathcal{I})^\alpha$ denote the set of x and y such that $x \overset{S_\theta^L(\mathcal{I})^\alpha}{\sim} y$.

Remark 2. For $\alpha=1$ the above definition coincides with asymptotically $\mathcal{I}$-lacunary statistical equivalent of multiple L. Further it must be noted in this context that asymptotically $\mathcal{I}$-lacunary statistical equivalent of order α to multiple L has not been studied till now. Obviously, if we take $\mathcal{I}=\mathcal{I}_{fin}$ asymptotically lacunary statistical equivalent of order α to multiple L is a s pecial case of asymptotically $\mathcal{I}$-lacunary statistical equivalent of order α to multiple L.

Theorem 1. Let $0<\alpha\le\beta\le 1$. Then $S(\mathcal{I})^\alpha \subset S(\mathcal{I})^\beta$.

Proof: Let $0<\alpha\le\beta\le 1$. Then

$$\frac{|\{k \le n : |\frac{x_k}{y_k} - L| \ge \varepsilon\}|}{n^\beta} \le \frac{|\{k \le n : |\frac{x_k}{y_k} - L| \ge \varepsilon\}|}{n^\alpha}$$

and so for any $\delta>0$,

$$\{n\in\mathbb{N}:\frac{|\{k\le n:|\frac{x_k}{y_k}-L|\ge\varepsilon\}|}{n^{\beta}}\ge\delta\}\subset$$
$$\{n\in\mathbb{N}:\frac{|\{k\le n:|\frac{x_k}{y_k}-L|\ge\varepsilon\}|}{n^{\alpha}}\ge\delta\}.$$

Hence if the set on the right hand side belongs to the ideal $\mathcal{I}$ then obviously the set on the left hand side also belongs to $\mathcal{I}$. This shows that $S(\mathcal{I})^{\alpha}\subset S(\mathcal{I})^{\beta}$.

Similarly we can show that

Theorem 2. Let $0<\alpha\le\beta\le 1$. Then

(i) $S_{\theta}^{L}(\mathcal{I})^{\alpha}\subset S_{\theta}^{L}(\mathcal{I})^{\beta}$.

(ii) In particular $S_{\theta}^{L}(\mathcal{I})^{\alpha}\subset S_{\theta}^{L}(\mathcal{I})$.

Definition 12. Let θ be a lacunary sequence; two number sequences $x=(x_k)$ and $y=(y_k)$ are strong $\mathcal{I}$-asymptotically lacunary equivalent of order α to multiple L provided that for any $\varepsilon>0$

$$\{r\in\mathbb{N}:\frac{1}{h_r^{\alpha}}\sum_{k\in I_r}|\frac{x_k}{y_k}-L|\ge\varepsilon\}\in\mathcal{I},$$

(denoted by $x\overset{N_{\theta}^{L}(I)^{\alpha}}{\sim}y$) and simply strong asymptotically $\mathcal{I}$-lacunary statistical equivalent of order α if $L=1$. Further, let $N_{\theta}^{L}(\mathcal{I})^{\alpha}$ denote the set of x and y such that $x\overset{N_{\theta}^{L}(\mathcal{I})^{\alpha}}{\sim}y$.

We prove the following

Theorem 3. Let $\theta=\{k_r\}_{r\in\mathbb{N}}$ be a lacunary sequence, then

(a) If $x\overset{N_{\theta}^{L}(\mathcal{I})^{\alpha}}{\sim}y$ then $x\overset{S_{\theta}^{L}(\mathcal{I})^{\alpha}}{\sim}y$

(b) $N_{\theta}^{L}(\mathcal{I})^{\alpha}$ is a proper subset of $S_{\theta}^{L}(\mathcal{I})^{\alpha}$

Proof: (a) If $\varepsilon>0$ and $x\overset{N_{\theta}^{L}(\mathcal{I})^{\alpha}}{\sim}y$, we can write

$$\sum_{k\in I_r}|\frac{x_k}{y_k}-L|\ge$$
$$\sum_{k\in I_r,|\frac{x_k}{y_k}-L|\ge\varepsilon}|\frac{x_k}{y_k}-L|\ge\varepsilon|\{k\in I_r:|\frac{x_k}{y_k}-L|\ge\varepsilon\}|$$

$$\textit{and so}\ \frac{1}{\varepsilon.h_r^{\alpha}}\sum_{k\in I_r}|\frac{x_k}{y_k}-L|\ge\frac{1}{h_r^{\alpha}}|\{k\in I_r:|\frac{x_k}{y_k}-L|\ge\varepsilon\}|.$$

Then for any $\delta>0$

$$\{r\in\mathbb{N}:\frac{1}{h_r^{\alpha}}|\{k\in I_r:|\frac{x_k}{y_k}-L|\ge\varepsilon\}|\ge\delta\}\subseteq$$
$$\{r\in\mathbb{N}:\frac{1}{h_r^{\alpha}}\sum_{k\in I_r}|\frac{x_k}{y_k}-L|\ge\varepsilon.\delta\}\in\mathcal{I}.$$

This proves the result.

(b) In order to establish that the inclusion $N_{\theta}^{L}(\mathcal{I})^{\alpha}\subseteq S_{\theta}^{L}(\mathcal{I})^{\alpha}$ is proper, let θ be given and define x_k to be $1,2,...,[\sqrt{h_r^{\alpha}}]$ at first $[\sqrt{h_r^{\alpha}}]$ integers in I_r and $x_k=0$ otherwise for all $r=1,2,3,...$. $y_k=1$ for all k. Then for any $\varepsilon>0$,

$$\frac{1}{h_r^{\alpha}}|\{k\in I_r:|\frac{x_k}{y_k}-0|\ge\varepsilon\}|\le\frac{[\sqrt{h_r^{\alpha}}]}{h_r^{\alpha}}$$

and for any $\delta>0$ we get

$$\{r\in\mathbb{N}:\frac{1}{h_r^{\alpha}}|\{k\in I_r:|\frac{x_k}{y_k}-0|\ge\varepsilon\}|\ge\delta\}\subseteq\{r\in\mathbb{N}:\frac{[\sqrt{h_r^{\alpha}}]}{h_r^{\alpha}}\ge\delta\}.$$

Since the set on the right hand side is a finite set and so belongs to $\mathcal{I}$ it follows that $x\overset{S_{\theta}^{L}(\mathcal{I})^{\alpha}}{\sim}y$.

On the other hand

$$\frac{1}{h_r^{\alpha}}\sum_{k\in I_r}|\frac{x_k}{y_k}-0|=\frac{1}{h_r^{\alpha}}.\frac{[\sqrt{h_r^{\alpha}}]([\sqrt{h_r^{\alpha}}]+1)}{2}.$$

Then

$$\{r\in\mathbb{N}:\frac{1}{h_r^{\alpha}}\sum_{k\in I_r}|\frac{x_k}{y_k}-0|\ge\frac{1}{4}\}=\{r\in\mathbb{N}:\frac{[\sqrt{h_r^{\alpha}}]([\sqrt{h_r^{\alpha}}]+1)}{h_r}\ge\frac{1}{2}\}$$
$$=\{m,m+1,m+2,...\}$$

for some $m\in\mathbb{N}$ which belongs to $F(\mathcal{I})$ since $\mathcal{I}$ is admissible. So the following fails $x\overset{N_{\theta}^{L}(\mathcal{I})^{\alpha}}{\sim}y$.

Remark 4. The following two conditions remain true for $0<\alpha<1$ is not clear and we leave them as open problems.

(2) $x\in l_{\infty}$ and $x\overset{S_{\theta}^{L}(\mathcal{I})^{\alpha}}{\sim}y\Rightarrow x\overset{N_{\theta}^{L}(\mathcal{I})^{\alpha}}{\sim}y$,

(3) $S_{\theta}^{L}(\mathcal{I})^{\alpha}\cap l_{\infty}=N_{\theta}^{L}(\mathcal{I})^{\alpha}\cap l_{\infty}$.

We now investigate the relationship between $x\overset{S^{L}(\mathcal{I})^{\alpha}}{\sim}y$ and $x\overset{S_{\theta}^{L}(\mathcal{I})^{\alpha}}{\sim}y$.

Theorem 4. Let $\mathcal{I}$ is an ideal and $\theta=\{k_r\}$ is a lacunary sequence, then

$x\overset{S^{L}(\mathcal{I})^{\alpha}}{\sim}y$ *implies* $x\overset{S_{\theta}^{L}(\mathcal{I})^{\alpha}}{\sim}y$

if $\lim\inf_r q_r^{\alpha}>1$.

Proof: Suppose first that $\lim\inf_r q_r^{\alpha}>1$. Then there exists $\sigma>0$ such that $q_r^{\alpha}\ge 1+\sigma$ for sufficiently large r which implies that

$$\frac{h_r^{\alpha}}{k_r^{\alpha}}\ge\frac{\sigma}{1+\sigma}.$$

Since $x\overset{S^{L}(\mathcal{I})^{\alpha}}{\sim}y$, then for every $\varepsilon>0$ and for sufficiently large r, we have

$$\frac{1}{k_r^{\alpha}}|\{k\le k_r:|\frac{x_k}{y_k}-L|\ge\varepsilon\}|\ge\frac{1}{k_r^{\alpha}}|\{k\in I_r:|\frac{x_k}{y_k}-L|\ge\varepsilon\}|$$
$$\ge\frac{\sigma}{1+\sigma}.\frac{1}{h_r^{\alpha}}|\{k\in I_r:|\frac{x_k}{y_k}-L|\ge\varepsilon\}|.$$

Then for any $\delta>0$, we get

$$\{r\in\mathbb{N}:\frac{1}{h_r^\alpha}|\{k\in I_r:|\frac{x_k}{y_k}-L|\geq\varepsilon\}|\geq\delta\}$$
$$\subseteq\{r\in\mathbb{N}:\frac{1}{k_r^\alpha}|\{k\leq k_r:|\frac{x_k}{y_k}-L|\geq\varepsilon\}|\geq\frac{\delta\sigma}{(1+\sigma)}\}\in\mathcal{I}.$$

This proves the result.

Remark 5. The converse of this result is not clear for $\alpha<1$ and we leave it as an open problem.

For the next result we assume that the lacunary sequence θ satisfies the condition that for any set $C\in F(\mathcal{I})$, $\bigcup\{n:k_{r-1}<n<k_r, r\in C\}\in F(\mathcal{I})$.

Theorem 5. For a lacunary sequence θ satisfying the above condition,

$$x\overset{S_\theta^L(\mathcal{I})^\alpha}{\sim}y \text{ implies } x\overset{S^L(\mathcal{I})^\alpha}{\sim}y$$

if $\sup_r\sum_{i=0}^{r-1}\frac{h_{i+1}^\alpha}{(k_{r-1})^\alpha}=B(say)<\infty.$

Proof: Suppose that $x\overset{S_\theta^L(\mathcal{I})^\alpha}{\sim}y$ and for $\varepsilon,\delta,\delta_1>0$ define the sets

$$C=\{r\in\mathbb{N}:\frac{1}{h_r^\alpha}|\{k\in I_r:|\frac{x_k}{y_k}-L|\geq\varepsilon\}|<\delta\}$$

and

$$T=\{n\in\mathbb{N}:\frac{1}{n^\alpha}|\{k\leq n:|\frac{x_k}{y_k}-L|\geq\varepsilon\}|<\delta_1\}.$$

It is obvious from our assumption that $C\in F(\mathcal{I})$, the filter associated with the ideal $\mathcal{I}$. Further observe that

$$A_j=\frac{1}{h_j^\alpha}|\{k\in I_j:|\frac{x_k}{y_k}-L|\geq\varepsilon\}|<\delta$$

for all $j\in C$. Let $n\in\mathbb{N}$ be such that $k_{r-1}<n<k_r$ for some $r\in C$. Now

$$\frac{1}{n^\alpha}|\{k\leq n:|\frac{x_k}{y_k}-L|\geq\varepsilon\}|\leq\frac{1}{k_{r-1}^\alpha}|\{k\leq k_r:|\frac{x_k}{y_k}-L|\geq\varepsilon\}|$$

$$=\frac{1}{k_{r-1}^\alpha}|\{k\in I_1:|\frac{x_k}{y_k}-L|\geq\varepsilon\}|+\ldots+\frac{1}{k_{r-1}^\alpha}|\{k\in I_r:|\frac{x_k}{y_k}-L|\geq\varepsilon\}|$$

$$=\frac{k_1^\alpha}{k_{r-1}^\alpha}\frac{1}{h_1^\alpha}|\{k\in I_1:|\frac{x_k}{y_k}-L|\geq\varepsilon\}|+\ldots+$$
$$\frac{(k_r-k_{r-1})^\alpha}{k_{r-1}^\alpha}\frac{1}{h_r^\alpha}|\{k\in I_r:|\frac{x_k}{y_k}-L|\geq\varepsilon\}|$$

$$=\frac{k_1^\alpha}{k_{r-1}^\alpha}A_1+\frac{(k_2-k_1)^\alpha}{k_{r-1}^\alpha}A_2+\ldots+\frac{(k_r-k_{r-1})^\alpha}{k_{r-1}^\alpha}A_r$$

$$\leq\sup_{j\in C}A_j.\sup_r\sum_{i=0}^{r-1}\frac{(k_{i+1}-k_i)^\alpha}{k_{r-1}^\alpha}<B\delta.$$

Choosing $\delta_1=\frac{\delta}{B}$ and in view of the fact that $\bigcup\{n:k_{r-1}<n<k_r,r\in C\}\subset T$ where $C\in F(\mathcal{I})$ it follows from our assumption on θ that the set T also belongs to $F(\mathcal{I})$ and this completes the proof of the theorem.

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