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Comparison of SPS and FA methods for Blind Carrier Frequency Offset Estimation in OFDM systems

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***Abstract*-In orthogonal frequency-division multiplexing (OFDM) systems, synchronization issues are of great importance since synchronization errors might destroy the orthogonality among all subcarriers and, therefore, introduce intercarrier interference (ICI) and intersymbol interference (ISI). Several schemes of frequency offset estimation in OFDM systems have been investigated. This paper compares performance and computational complexity of Smoothing Power Spectrum (SPS) and Frequency Analysis (FA) methods for blind carrier frequency offset (CFO) estimation in OFDM systems.**

***Key words* – SPS method, FA method, Carrier Frequency Offset estimation, OFDM**

I. INTRODUCTION

OFDM represents an efficient technique distinguished for high-speed digital transmission over multipath fading channels. It is well known that OFDM provides a fast developing technology for the next generation of mobile communications, however beside the inherent defects such as time-synchronous error and inter-carrier interference within OFDM, high sensitivity to carrier frequency offset (CFO) has been widely recognized as its considerable weakness. In practical OFDM systems, the existence of CFO, which is caused due to mismatch between the oscillator in the transmitter and the receiver, destroys orthogonality among OFDM subcarriers and leads to performance degradation

In order to mitigate this effect, various techniques have been proposed to estimate the CFO for OFDM systems [3]–[12]. In [3], Moose proposed a maximum likelihood (ML) estimator using repeated data symbol. Although the symbol values need not be known, repetition of OFDM symbols is in essence tantamount to a training-based scheme as it utilizes extra bandwidth. Data-assisted frequency acquisition and tracking were proposed in [4], where periodically inserted known symbols were explicitly used. In [5], Schmidl and Cox proposed a training symbol-based timing/frequency synchronization that utilized an OFDM symbol with identical halves. This was later generalized to a training symbol with multiple identical parts [9]. Various blind techniques have also been proposed. In [8], van de Beek *et al.* developed an ML estimator by exploiting the redundancy in the cyclic prefix. This method, however, is developed based on the nondispersive channel model and suffers the error floor effect in the presence of frequency-selective fading channels. Schmidl and Cox proposed in [9] a blind estimation method that is only suitable to recover CFO values that are multiples of the carrier spacing.

In [10], Choi proposed an ML estimator by assuming that the OFDM signal is complex Gaussian distributed, which is asymptotically true for circularly modulated (CM) OFDM symbols [13], [14]. The proposed algorithm, however, requires perfect knowledge of the second-order statistics about the channel noise and the Rayleigh fading channel. In [11] and [12], Liu and Tureli took advantage of the presence of virtual carriers in OFDM signaling and proposed blind estimation methods reminiscent of spectral analysis techniques in array processing, i.e., MUSIC and ESPRIT. It was later shown that the proposed MUSIC algorithm is indeed the ML estimate of the CFO with a virtual carrier present signal model [15]–[17]. The drawback is that it usually requires multiple OFDM symbols to achieve desirable performance, thus introducing an extra delay at the receiver. Receiver diversity-based CFO estimation was given in [18], where multiple receive antennas were employed. Again, the presence of virtual carriers was exploited and the proposed estimator is indeed an extension of the MUSIC algorithm much in the same way as its extension to the case when multiple OFDM blocks are used with a single antenna.

In [21], a blind CFO estimation method was proposed in terms of a kurtosis based cost function. The blind method was simplified by approximating the kurtosis cost function as a sinusoidal function. However, to express the kurtosis based cost function as a sinusoidal function, more than 10 OFDM symbols should be collected. In [22], the CFO was blindly estimated such that the ICI was statistically minimized in the collected hundreds of OFDM symbol blocks. However, the method is not desirable since it takes long time to collect hundreds of OFDM symbol blocks.

In [1] Lu Wu, Xian-Da Zhang, Pei-Sheng Li and Yong-Tao Su proposed a blind CFO estimator based on Smoothing Power Spectrum (SPS) which outperformed CP[24], the Kurtosis[26] and Variance (VE)[28] methods . In [2] Lu Wu, Xian-Da Zhang, Pei-Sheng Li and Yong-Tao Su proposed a blind CFO estimator based on Frequency Analysis(FA) which outperformed CP[24], MOV[25] and Kurtosis [26] methods and its performance is equivalent with subspace (SBS) method[27] but computational complexity of FA method is much less than SBS method.

II. OFDM TRANSMITTER

Consider an OFDM system with N subcarriers. The transmitted data in the i^{th} block, defined as $\mathbf{s}(i) = [s_0(i),\ s_1(i),\ \cdot\cdot\cdot\ ,\ s_{N-1}(i)]^T$, are supposed to be independent and circularly symmetric with zero mean and variance σ_S^2. They are then modulated onto the subcarriers by inverse discrete Fourier transform (IDFT). The CP (Cyclic Prefix) is inserted to avoid the intersymbol interference, with the length P no less than the channel order L. A frequency-selective block fading channel is assumed. $h(i) = [h_0(i),\ h_1(i),\ \cdot\cdot\cdot\ ,\ h_L(i)]^T$ denotes the channel impulse response, corresponding to the channel frequency response $H(i) = [H_0(i), H_1(i), \ldots, H_{N-1}(i)]^T$.

2.1 SPS (Smoothing Power Spectrum) Method

At the receiver, the CP is first removed, and then the received signal is compensated by the CFO candidate v and demodulated by discrete Fourier transform (DFT). So the resulting signal on the k^{th} subcarrier can be written as

$$y_v^k(i) = U_{kk}(\delta)H_k(i)S_k(i) + \sum_{n\neq k} U_{kn}(\delta)H_n(i)S_n(i) + w_k(i) \quad \text{----- (1)}$$

Let $\varepsilon \in (-0.5,\ 0.5]$ represent the CFO (carrier Frequency Offset) normalized by the intercarrier spacing and $\delta = \varepsilon - v$ is CFO estimation error. $U_{mn}(\delta)$ is the $(m,\ n)^{th}$ element of $\phi_\delta(i) F \Lambda(\delta) F^H$, $\Lambda(\delta) \triangleq \text{diag}[1, e^{j2\pi\delta/N}, \ldots\ldots, e^{j2\pi\delta(N-1)/N}]$, $\phi_\delta(i) \triangleq e^{j\frac{2\Pi}{N}\delta[i(N+P)+P]}$,

$F \triangleq \{\frac{1}{\sqrt{N}} e^{-j\frac{2\pi}{N}mn}\}_{m,n=0}^{N-1}$ denotes DFT matrix, and $w_k(i) \sim CN(0, \sigma_n^2)$ is independent noise. The second term of (1) represents ICI (inter carrier interference). Using i^{th} OFDM block, signal power spectrum can be given by

$$p_{r,v}^k = \frac{1}{N} | y_v^k(i) |^2 \quad \text{---- (2)}$$

Channel power spectrum can be given by

$$p_h^{k+1} = \frac{1}{N} | H_{k+1}(i) |^2$$

$$= \frac{1}{N} \sum_{n=0}^{L} \sum_{l=0}^{L} h_n(i) h_l^*(i) e^{-j\frac{2\pi k}{N}(n-l)} e^{-j2\pi\frac{n-l}{N}}$$

When $(n-l)/N \in [-L/N, L/N] \rightarrow 0$ as proposed in [1] if channel order is greatly less than the block size , $p_h^{k+1}(i) \approx p_h^k(i)$ indicates that the channel power spectrum is smooth when L << N. Signal power spectrum without CFO compensation fluctuates much more than the one with perfect CFO compensation.

To measure fluctuation of $p_{r,v}^k(i)$, we can make use of the difference between adjacent subcarriers and form the following objective function

$$J(v) \triangleq \frac{1}{M} \sum_{k=0}^{N-1} \sum_{i=0}^{M-1} (p_{r,v}^k(i) - p_{r,v}^{k+1}(i))^2 \quad \text{----- (3)}$$

Where M is number of OFDM blocks, $p_{r,v}^{N}(i) \triangleq p_{r,v}^{0}(i)$. When M is large and L<<N, it can be given that

$$J(v) \approx A\cos(2\pi\delta) + B \quad \text{----- (4)}$$

Where

A=

$$\frac{2\sigma^4}{N^4}\sum_{m=0}^{N-1}\left[\frac{(K_S-1)(N^2-1)_{\rho m,m}}{3} - \frac{(2-K_S)_{\rho m,m}}{\sin^2\frac{\pi}{N}} - \sum_{a\neq m}\frac{\rho a,m}{\sin^2\frac{(a-m)\pi}{N}}\right]$$

$$\rho_{x,y} \triangleq E[|H_x(i)H_y(i)|^2] \quad ,$$

$$K_S \triangleq E[|S_K(i)|^4]/\sigma_S^4$$

B is a constant independent of δ, (4) shows that fluctuations of $p_{r,v}^{k}(i)$ has the form of cosine about δ and $\delta = 0$ is a global minimum when $A < 0$ and a global maximum when $A > 0$, and indicates the sign of A such that

If $K_S < 1 + \frac{9}{3+\pi^2}$, $A < 0$ holds

1. For super-Gaussian signals ($K_S > 2$), $A > 0$ always holds.
2. For sub-Gaussian signals ($K_S < 2$), $A < 0$ exists if the channel power spectrum does not change fast ($L << N$).
3. For Gaussian signals ($K_S = 2$), if the channel is frequency selective, $A > 0$ holds.

Considering the cosine form in (4), a closed-form CFO estimate can be obtained by employing parameter estimation. By evaluating $J(v)$ at three special points, e.g. v = -1/4,0,1/2 denoted by J_1, J_2, J_3 respectively, and trying to find ε in (4), we can get the CFO estimate as

$$\hat{\varepsilon} = \begin{cases} \theta/2\pi & c \geq 0, A<0 \;\; or \;\; c\leq 0, A>0 \\ \theta/2\pi + 0.5 & c<0, d\geq 0 \;\; or \;\; c>0, d\leq 0, A>0 \\ \theta/2\pi - 0.5 & c<0, d<0 \;\; or \;\; c>0, d>0, A>0 \end{cases} \quad \text{--(5)}$$

Where $\theta \triangleq \arctan(d/c)$, c$\triangleq$(J_3-J_2)/2 and d$\triangleq$$J_1$-($J_2$+$J_3$)/2

2. 2. FA (Frequency Analysis) Method

At the receiver CP is removed and received symbol denoted by $r(i)=[r_0(i),r_1(i),\ldots r_{N-1}(i)]^T$, is compensated by the CFO candidate u , demodulated by DFT (Discrete Fourier Transform) and the resulting symbol can be written as

$$X_u(i) = F\Lambda_N(-u)r(i) \text{----- (6)}$$

$$= e^{j\theta_\varepsilon(i)}F\Lambda_N(\bar{\varepsilon})F^H D_H(i)S(i) + W(i) \text{----- (7)}$$

Where $\varepsilon \in$(-0.5, 0.5] represents CFO, $\bar{\varepsilon} = \varepsilon - u$ is the estimation error, $\theta_\varepsilon(i) = 2\pi\varepsilon$ [$i(N+P)+P]/N$, $\Lambda_N(x) = diag(1, e^{j2\Pi x/N}, \ldots\ldots, e^{j2\Pi x(N-1)/N})$. $D_H(i) = diag(H_0(i), H_1(i), \ldots\ldots, H_{N-1}(i))$, is a diagonal matrix where $H_k(i) = \sum_{l=0}^{L} h_l(i)e^{-j2\Pi kl/N}$,

$[F]_{mn} = \frac{1}{\sqrt{N}} e^{-j2\Pi mn/N}$ represents N-point DFT matrix, and $W(i) \sim CN(0, \sigma_n^2 I)$ denotes noise where I is identity matrix. If CFO is estimated perfectly (if $\varepsilon = u$) (7) can be given as

$$X_\varepsilon(i) = e^{j\theta_\varepsilon(i)}D_H(i)S(i) + W(i) \quad \text{----(8)}$$

As proposed in [2] with CM constellations if $|S_k(i)|$=1 from (8)

$$[X_\varepsilon(i) \odot X_\varepsilon^*(i)]_n \; - = \sum_{l_1=0}^{L} h_{l_1}(i)e^{-j2\Pi nl_1/N}\sum_{l_2=0}^{L} h_{l_2}^*(i)e^{j2\Pi nl_2/N}$$

$$= \sum_{l=-L}^{L} pl(i)e^{-j2\Pi l/N} \quad \text{----- (9)}$$

Where $\odot$ represents Hadamard product, X_k represents k^{th} element of X and

$$pl(i) = \sum_{t=\max(0,l)}^{\min(L,L+l)} h_t(i)h_{t-l}^*(i)$$

In case of imperfect CFO compensation, only 2N+1components present in $[X_\varepsilon(i) \odot X_\varepsilon^*(i)]_n$. Further if is $y_\varepsilon(i)$ defined $F^H(X_\varepsilon(i) \odot X_\varepsilon^*(i))$. From (8), k^{th} element of $y_\varepsilon(i)$ can be given as

$$|y_\varepsilon(i)|_k = \frac{1}{\sqrt{N}}\sum_{l=-L}^{L} pl(i)\sum_{n=0}^{N-1} e^{j2\Pi n(k-l)/N}$$

$$= \begin{cases} \sqrt{N_{pk}}(i) & 0 \leq k \leq L \\ 0 & L+1 \leq k \leq N-L-1 \\ \sqrt{N_{pk-N}(i)} & N-L \leq k \leq N-1 \end{cases} \text{--- (10)}$$

Since $[F^H X]_k = [FX]_{N-k}$, it can be given from (9) that $[F(X_\varepsilon(i) \odot X_\varepsilon^*(i))]_k = 0$ for $L+1 \le k \le N-L-1$, which implies that to compensate CFO completely, certain frequencies of received signal disappear. Let $y_u(i) = [y_u^0(i), y_u^1(i), \ldots\ldots, y_u^{N-1}(i)]^T = F^H(X_\varepsilon(i) \odot X_\varepsilon^*(i))$. Based on the analysis give above, CFO can be estimated by minimizing the sum of $|y_u^k(i)|^2$ for $L+1 \le k \le N-L-1$. Thus in [2] the following Frequency Analysis (FA) was proposed

$$J_{FA}(u) = \sum_{k=L+1}^{N-L-1} |y_u^k(i)|^2 \quad \text{----- (11)}$$

$$\hat{\varepsilon} = \arg\min_u J_{FA}(u) \quad \text{----- (12)}$$

2.2.1 Closed form CFO estimator

Closed solution of (12) reduces the computational complexity of CFO estimator proposed in 2.1. In (6) time index is omitted for simplicity then it yields

$$|X_u \odot X_u^*|_n = \frac{1}{N}\sum_{k_1=0}^{N-1}\sum_{k_2=0}^{N-1} e^{-j2\Pi u(k_1-k_2)/N} \times rk_1 r^* k_2 e^{-j2\Pi N(k_1-k_2)/N}$$

$$= \frac{1}{\sqrt{N}} \sum_{k=-N+1}^{N-1} \hat{R}(k) e^{-j2\Pi nk/N} \quad \text{----- (13)}$$

Where $k = k_1 - k_2$ and

$$\hat{R}_u(k) = \frac{1}{\sqrt{N}} e^{-j2\Pi uk/N} \sum_{t=\max\{0,-k\}}^{\min\{N-K-1,N-1\}} r_t + kr_t^* \quad \text{-- (14)}$$

Since $y_u = F^H(X_\varepsilon(i) \odot X_\varepsilon^*(i))$ from (12) it can be given that $y_u^0 = R_u(0)$ and

$$y_u^k = \hat{R}_u(k) + \hat{R}_u(k-N) \;,\; 1 \le k \le N-1 \quad \text{-(15)}$$

Let $a_k = |\hat{R}_u(k)| = \frac{1}{\sqrt{N}} \left| \sum_{t=0}^{N\_k-1} r_t + kr_t^* \right|$

$$\theta_k = angle(\sum_{t=N-k}^{N-1} r_t + k - Nr_t^* - angle(\sum_{t=0}^{N-k-1} r_t + kr_t^*)$$

Where angle (.) represents phase of complex number moreover a_k, b_k and θ_k are independent of u. According to (14) it can be given that

$$angle(\hat{R}_u(k-N)) - angle(\hat{R}_u(k)) = 2\pi u + \theta_k$$

. Due to (15) following equation can be obtained

$$|y_u^k|^2 = a_k^2 + b_k^2 + 2a_k b_k \cos(2\pi u + \theta_k),$$

$$1 \le k \le N-1 \quad \text{----- (16)}$$

Substituting (16) in (11) results in

$$J_{FA}(u) = \sum_{k=L+1}^{N-L-1} a_k^2 + b_k^2 + 2a_k b_k \cos(2\pi u + \theta_k)$$

$$= C_1 + C_2 \cos(2\pi u + C_3) \quad \text{----- (17)}$$

Where C_1, C2 and C_3 are all constants independent of u. J_{FA}(u) is a cosine function which reaches minimum with accurate CFO estimation. Due to cosine form of (17) we can evaluate at three special points (i.e. 0,1/4,1/2) and find C_1, C2 and C_3. The proposed CFO estimate in (12) can be given as

$$\hat{\varepsilon} = \begin{cases} -\frac{1}{2\pi}\tan^{-1}(a/b) & if \;\; b \ge 0 \\ -\frac{1}{2} - \frac{1}{2\pi}\tan^{-1}(a/b) & if \;\; a > 0 \;\; and \;\; b<0 \\ \frac{1}{2} - \frac{1}{2\pi}\tan^{-1}(a/b) & if \;\; a \le 0 \;\; and \;\; b<0 \end{cases} \quad \text{- (18)}$$

III. PERFORMANCE ANALYSIS

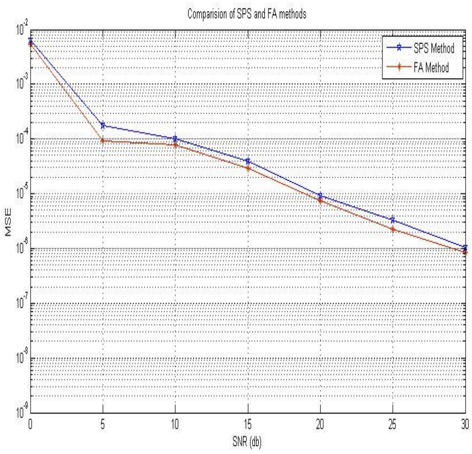

Fig 1 Comparison of SPS and FA methods

In this section, performance of SPS method is compared with FA method in an OFDM system with QPSK constellation, $N = 64$ and $P = 5$. A six-tap Rayleigh block fading channel is utilized with the exponentially decaying powers set as $E[|h_n(i)|^2] = \frac{e^{-n/(L+1)}}{\sum_{m=0}^{L} e^{-m/(L+1)}}$, n=0, 1, 2….., L [23]. . The CFO is assumed to be uniformly distributed between (-0.5, 0.5]. All results are averaged by 5000 Monte Carlo trials. Fig. 1 shows the MSE (mean square errors) of SPS and FA methods. From Fig1 it can be seen that FA method performs better than SPS method. Moreover computational complexity of FA method is less than SPS method. But FA method should be used for systems only with CM constellations whereas SPS method can be used for systems having CM constellations and non-CM constellations.

IV. CONCLUSION

This paper compares SPS and FA methods for blind CFO estimation for OFDM system. Computational complexity of FA method is less compared to SPS method and simulation results shown that FA method performs better than SPS method. But FA method should be used for systems only with CM constellations whereas SPS method can be used for systems having CM constellations and non-CM constellations.

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