Hall effect on MHD flow and heat transfer over an unsteady stretching permeable surface in the presence of thermal radiation and a heat source/sink

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Abstract—This paper employs the computational iterative approach known as Spectral Local Linearization Method (SLLM) to analyze Hall effect on MHD flow and heat transfer over an unsteady stretching permeable surface in the presence of thermal radiation and heat source/sink. To demonstrate the reliability of our proposed method, we made comparison with Matlab bvp4c routine technique and excellent agreement was observed. The governing partial equations are transformed into a system of ordinary differential equations by using suitable similarity transformations. The results are obtained for velocity, temperature, skin friction and Nusselt number.

Keywords—Hall effect;stretching sheet ; thermal radiation, heat source/sink.

I. Introduction

Theoretical studies of magnetohydrodynamic flow and heat transfer over stretching surfaces have received great attention by virtue of their numerous applications in the fields of metallurgy, chemical engineering and biological systems. Such applications include geothermal reservoirs, wire and fiber coating, food stuff processing, reactor fluidization, enhanced oil recovery, packed bed catalytic reactors, and cooling of nuclear reactors. The primary aim in extrusion is to maintain the quality of the surface of the extricate.

Examples of such studies include Sakiadis[1], [2] did pioneering work on boundary layer flow on a continuously moving surface. Shateyi and Motsa [3] carried out a numerical analysis of the problem of magnetohydrodynamic boundary layer flow, heat and mass transfer rates on steady two-dimensional flow of an electrically conducting fluid over a stretching sheet embedded in a non-Darcy porous medium in the presence of thermal radiation and viscous dissipation. Shateyi [4] investigated thermal radiation and buoyancy effects on heat and mass transfer over a semi-infinite stretching surface with suction and blowing. Singh et al. [5] investigated two dimensional unsteady flow of a viscous incompressible fluid about a stagnation point on a permeable stretching sheet. Shateyi and Motsa [6] numerically investigated the unsteady heat, mass and fluid transfer over a horizontal stretching sheet. More recently, Shateyi and Marewo [7] studied the magnetohydrodynamic boundary layer flow with heat and mass transfer of an UCM fluid over a stretching sheet in the presence of viscous dissipation and thermal radiation.

Governing equations modeling MHD flow and heat transfer over stretching surfaces are highly nonlinear thereby exact solutions are impossible to obtain. Therefore, numerical solutions have always been developed and modified, as a bid of getting more accurate and stable solutions. The current study seeks to investigate the Hall effect on MHD flow and heat transfer over an unsteady stretching permeable surface in the presence of thermal radiation and a heat source/sink. We propose to numerically solve the present problem using a recently developed iterative method known as Spectral Local Linearization Method (SLLM), Motsa [8]. The SLLM approach is based on transforming nonlinear ordinary differential equation into an iterative scheme. The iterative scheme is then blended with Chebyshev spectral method ([9]).

II. Model Formulation

We consider an unsteady two-dimensional laminar MHD boundary layer flow and heat transfer of an incompressible, viscous and electrically conducting fluid over a continuously moving stretching permeable surface.

The flow is subjected to a transverse magnetic field of strength B_0 and the Hall current is taken into account in this study.

The relevant governing equations of fluid flow and heat transfer are,

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0, \tag{1}$$

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\nu\frac{\partial^2 u}{\partial y^2}-\frac{\sigma B^2}{\rho(1+m^2)}(u+mw), \tag{2}$$

$$\frac{\partial w}{\partial t}+u\frac{\partial w}{\partial x}+v\frac{\partial w}{\partial y}=\nu\frac{\partial^2 w}{\partial y^2}-\frac{\sigma B^2}{\rho(1+m^2)}(mu-w), \tag{3}$$

$$\frac{\partial T}{\partial t}+u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 T}{\partial y^2}-\frac{1}{\rho c_p}\frac{\partial q_r}{\partial y}+\frac{Q}{\rho c_p}(T-T_\infty). \tag{4}$$

The associated boundary conditions to the current problem are:

$$\begin{aligned} &u=U_w(x,t),\ v=V_w,\ T=T_w(x,t), \quad \text{at} \quad y=0,\\ &u\to 0,\ w\to 0,\ T\to T_\infty \quad \text{as} \quad y\to\infty, \end{aligned} \tag{5}$$

where $u,\ v$ and w are the velocity components along $x,\ y$ and z directions, respectively and t is the time. T is the temperature within the fluid, c_p is the specific heat at constant pressure, α is the thermal diffusivity, ν is the kinematic viscosity of the fluid density, $T_w(x,t)$ is the temperature on the stretching surface, T_∞ is the ambient temperature with $T_w > T_\infty$. We have $V_w=-(\nu U_w/x)^{1/2}f(0)$ representing the mass transfer at the surface with $V_w>0$ for injection and $V_w<0$ for suction. We also have $U_w(x,t)=ax/(1-ct)$, where a (stretching rate) and c are positive constants, with $ct<1$. It is noted that the stretching rate $a/(1-ct)$ increases with time since $a>0$. The surface temperature of the sheet varies with the distance x from the origin and time t and takes the form:

$$T_w(x,t)=T_\infty+\frac{b^2x}{2\nu(1-ct)^{3/2}}, \tag{6}$$

where b is a constant with $b\geq 0$.

A. Similarity Transformation

Following Ishak et al. [10], we introduce the following dimensionless functions f and θ, and the similarity variable η.

$$\begin{aligned} &\eta=\left(\frac{b}{\nu(1-ct)}\right)^{\frac{1}{2}}y,\ \psi(x,y,t)=\left(\frac{\nu b}{1-ct}\right)^{\frac{1}{2}}xf(\eta),\\ &T(x,y,t)=T_\infty+\frac{b^2x}{2\nu(1-ct)^{\frac{3}{2}}}\theta(\eta), \quad B^2=\frac{B_0^2}{(1-ct)}. \end{aligned} \tag{7}$$

By using the Rosseland approximation, the radiative heat flux is given by

$$q_r=-\frac{4\sigma^*}{3K^*}\frac{\partial T^4}{\partial y}, \tag{8}$$

where $\sigma*$ and K^* are respectively, the Stephan-Boltzman constant and the mean absorption coefficient. Assuming that the temperature differences within the flow are such that T^4 can be expressed as a linear function. Expanding T^4 in a Taylor series about T_∞ and neglecting higher order terms we get

$$T^4\cong 4T_\infty T-3T_\infty^4. \tag{9}$$

By using the above transformations, the governing partial differential equations are transformed into a system of non-dimensional nonlinear and coupled ordinary differential equations as follows:

$$f'''+ff''-f'^2-A(f'+\frac{1}{2}\eta f'')-\frac{M}{1+m^2}(f'+mg)=0, \tag{10}$$

$$g''+fg'-f'g-A(g+\frac{1}{2}\eta g')+\frac{M}{1+m^2}(mf'-g)=0, \tag{11}$$

$$\left(1+\frac{4}{3}R\right)\theta''+Pr(f\theta'-2f'\theta)-Pr\frac{A}{2}(3\theta+\eta\theta')+\delta\theta=0, \tag{12}$$

Here $M^2=\sigma B_0^2/\rho a$, $A=c/a$ is a parameter that measures the unsteadiness, $Pr=\nu/\alpha$ is the Prandtl number, $R=4\sigma^*T_\infty^3/kK_s$ is the thermal radiation parameter. The boundary conditions are

$$f(0)=f_w, \quad f'(0)=1,\ \theta(0)=1\ g(0)=0, \tag{13}$$

$$f'\to 0, \quad \theta\to 0, \quad g\to 0, \quad \text{as} \quad \eta\to\infty, \tag{14}$$

where $f(0)=f_w$ with $f_w<0$ or $f_w>0$ corresponding to injection or suction, respectively. The physical engineering quantities of interest in this problem are the skin friction coefficients in the $x-$ and $z-$ directions and the local Nusselt number number, Nu_x which are defined as:

$$C_{fx} = -\frac{2\mu(\partial u/\partial y)_{y=0}}{\rho U_w^2} = -2Re_x^{-1/2} f''(0),$$
$$C_{fz} = \frac{2\mu(\partial w/\partial y)_{y=0}}{\rho U_w^2} = 2Re_x^{-1/2} g'(0),$$
$$Nu_x = \frac{xq_w}{\kappa(Tw - T_\infty)}, \quad (15)$$

where $\tau_w = \mu \left(\frac{\partial u}{\partial y}\right)_{y=0}$ is the wall shear stress, and $q_w = -\kappa \left(\frac{\partial T}{\partial y}\right)_{y=0}$ is the surface heat flux, where μ and κ are the dynamic viscosity and thermal conductivity, respectively.

III. Method of solution

If we let $f' = p$ then equations (10-12) become

$$p'' + fp' - p^2 - A\left(p + \frac{\eta}{2}p'\right) - \frac{M}{1+m^2}(p + mg) = 0 \quad (16)$$

$$g'' + fg' - pg - A\left(g + \frac{\eta}{2}g'\right) + \frac{M}{1+m^2}(mp - g) = 0 \quad (17)$$

$$\left(1 + \frac{4}{3}R\right)\theta'' + Pr(f\theta' - 2p\theta) - Pr\frac{A}{2}(3\theta + \eta\theta') + \delta\theta = 0$$

Equations (16-18) together with the change of variable $f' = p$ may be written as

$$L_1 + N_1 = H_1 \quad (18)$$
$$L_2 + N_2 = H_2 \quad (19)$$
$$L_3 + N_3 = H_3 \quad (20)$$
$$L_4 + N_4 = H_4 \quad (21)$$

IV. Results and Discussion

The numerically results iteratively generated by the SLLM for the main parameters that have significant effects on the flow properties are presented in this section. All the SLLM results presented in this work were obtained using $N = 50$ collocation points, and were are glad to high light that convergence was achieved as few as six iterations. We take the infinity value η_∞ to be 40. The magnetic field is taken quite strong by assigning large values of M to ensure generation of Hall currents. Unless otherwise stated, the default values for the parameters are taken as $M = 1,\ Pr = 0.71,\ \sigma = 0.5,\ R = 1,\ m = 0.5,\ f_w = 1$. In order to validate our numerical method, it was compared to MATLAB routine *bvp4c* which is an adaptive Lobatto quadrature iterative scheme.

In Table I we present a comparison between the SLLM approximate results and the *bvp4c* results for selected default values of the stretching parameter A. It can be clearly seen from this table that there is an excellent agreement between the results from the two methods. Also Table I shows that an increase in the unsteadiness parameter leads to increases in the skin-friction coefficients in both directions. Also the heat transfer gradient increases as the values of the unsteadiness parameter increase. The negative values of $f''(0)$ mean that the solid surface exerts a drag force on the fluid. This is due to the development of the velocity boundary layer which in the current study is caused solely by the stretching sheet. In Table II we display the effect of the magnetic parameter on the skin friction coefficients and the Nusselt number. The magnetic parameter M represents the significance of the magnetic field on the flow properties. As the magnetic strength increases, the dragging effect is clearly seen by the significant increments in the skin friction. We also observe that increasing the values of the Hartman number leads to the lowering of the values of the Nusselt number. Application of a strong magnetic field reduces the velocity which in turn increases heat diffusion within the fluid flow. This physically explains why heat transfer at the wall is reduced as M is increased.

TABLE I
Comparison of the SLLM results of $-f''(0),\ g'(0),\ -\theta'(0)$ with those obtained by *bvp4c* for different values of the unsteadiness parameter.

	$-f''(0)$		$g'(0)$		$-\theta'(0)$	
A	*bvp4c*	SLLM	*bvp4c*	SLLM	*bvp4c*	SLLM
1	2.06334	2.06334	0.17552	0.17552	0.95974	0.95974
2	2.27278	2.27278	0.15185	0.15185	1.30759	1.30759
3	2.46650	2.46650	0.134598	0.134598	1.54422	1.54422

TABLE II
Comparison of the SLLM results of $-f''(0),\ g'(0),\ -\theta'(0)$ with those obtained by *bvp4c* for different values of the magnetic parameter.

	$-f''(0)$		$g'(0)$		$-\theta'(0)$	
M	*bvp4c*	SLLM	*bvp4c*	SLLM	*bvp4c*	SLLM
1	2.06334	2.06334	0.17552	0.17552	0.51730	0.51730
3	2.40060	2.40060	0.41758	0.41758	0.46088	0.46088
5	2.69188	2.69188	0.59380	0.59380	0.43117	0.43117

Table III displays the influence of the Hall current on the skin friction coefficients as well as the Nusselt number. The skin friction coefficient is reduced as the values of the Hall current parameter increase. This explains why the skin friction coefficient in the axial direction. However, in the transverse direction the skin friction increases as the Hall current increases. There is slight effect of the Hall current on Heat transfer rate on the stretching surface. The influence of suction/injection parameter f_w on the axial velocity is depicted in Figure 1. The axial velocity is significantly influenced by this parameter. The velocity boundary layer is greatly enhanced when fluid is injected $(f_w < 0)$ into the flow system thereby increasing the velocity profiles. However, removing fluid from the flow system through suction, as expected drastically reduces the velocity profiles as can be clearly seen in Figure 1.

TABLE III
COMPARISON OF THE SLLM RESULTS OF $-f''(0),\ g'(0),\ -\theta'(0)$ WITH THOSE OBTAINED BY $bvp4c$ FOR DIFFERENT VALUES OF THE HALL PARAMETER.

	$-f''(0)$		$g'(0)$		$-\theta'(0)$	
m	$bvp4c$	SLLM	$bvp4c$	SLLM	$bvp4c$	SLLM
0.1	2.2059	2.2059	0.0312	0.0312	0.4978	0.4978
0.5	2.1537	2.1537	0.1311	0.1311	0.5041	0.5041
1.0	2.0633	2.0633	0.1755	0.1755	0.5173	0.5173

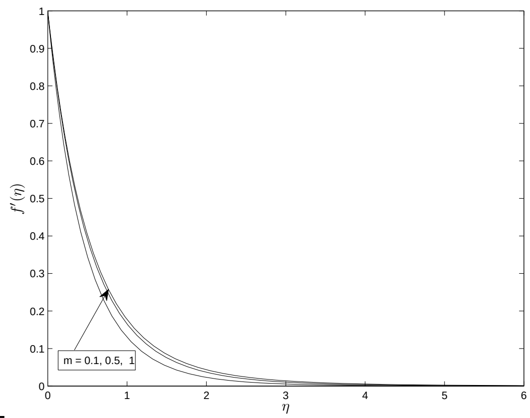

Fig. 1. Graph of the SLLM solutions for the horizontal velocity for different values of f_w.

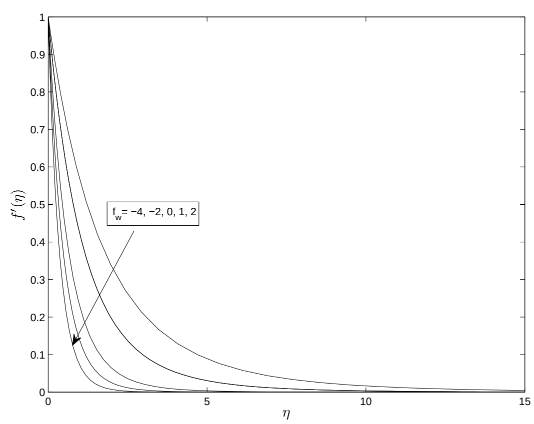

Fig. 2. Graph of the SLLM solutions for the horizontal velocity for different values of Hall parameter

The effect of the Hall current parameter m on the axial velocity is shown in Figure 2. The velocity is enhanced as the values of m increase. However, the axial velocity profiles approach their classical values when the Hall current parameter m becomes large $(m > 1.5)$ in our current study. Any further increase of the Hall current would make the magnetic effect insignificant.

The effect of the Hall current parameter on the transverse velocity is displayed in Figure 3. Increasing the values of m causes the transverse velocity to rapidly increase.

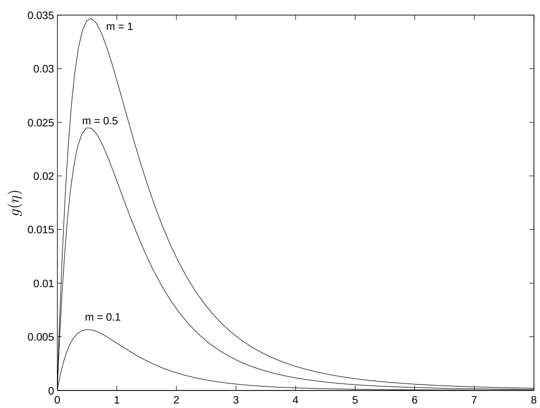

Fig. 3. Graph of the SLLM solutions for the transverse velocity for different values of the Hall parameter

In Figure 4 we have the effect of the heat source/sink parameter δ on the temperature profiles. As expected, it is observed in this figure that the temperature in the boundary layer increases with increasing values of δ. The heat absorption due to a uniform sink $(\delta < 0)$ leads to the reduction of the thermal boundary layer thickness, whereas this layer increases significantly with increases in $\delta > 0$.

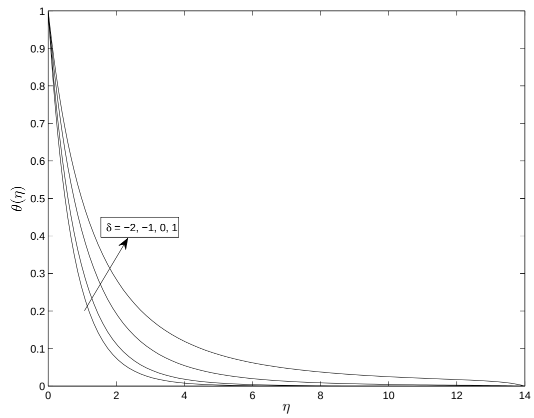

Fig. 4. Graph of the SLLM solutions of the temperature profiles for different values of heat source/sink parameter

Figure 5 is plotted to depict the influence of the thermal radiation parameter R on the temperature profiles. We clearly observe that the temperature in the boundary layer increases with increasing values of the thermal radiation parameter. This is due to the fact that the divergence of the radiative heat flux increases as the Rosseland radiative absorption K^* decreases which in turn increases the rate of radiative heat transfer to the fluid. Thus the presence of thermal radiation enhances thermal state of the fluid causing its temperature to significantly increase.

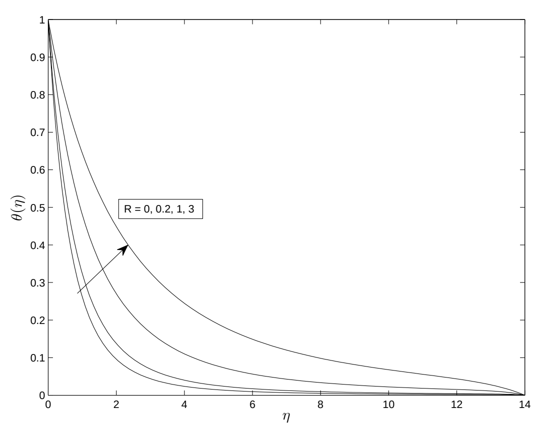

Fig. 5. Graph of the SLLM solutions of the temperature profiles for different values of thermal radiation parameter

V. Conclusion

The present work analyzed MHD unsteady flow and heat transfer of an electrically conducting fluid over a stretching sheet in the presence of thermal radiation and Hall effect. The governing partial differential equations are transformed into a system of non-linear ordinary differential equations by using suitable similarity variables. The resultant system of non-linear ordinary differential equations is solved numerically by the recently developed technique known as the Spectral Local Linearization Method. The accuracy of the SLLM is validated against the MATLAB in-built $bvp4c$ routine for solving boundary value problems. The following conclusions were drawn in our investigation.

- An excellent agreement was observed between our results and those obtained using $bvp4c$ routine technique giving confidence to our present results.
- The unsteadiness parameter A has significant effects on the velocity components and temperature profiles. The maximum axial velocity, transverse velocity and temperature profiles are attained when the flow is steady $(A = 0)$.
- Increasing the values of the magnetic field strength decreases the momentum boundary layer thickness while increasing the thermal boundary layer thickness.
- The velocity components are enhanced as the Hall parameter increases.
- The fluid temperature increases with increasing values of thermal radiation as well as a heat source.
- The heat transfer rate and the skin friction coefficient in the $x-$ direction are increased while the skin friction in the $z-$ direction decreases as the unsteadiness parameter increases.
- The skin friction coefficients are enhanced while the heat transfer rate is depressed by increasing values of the magnetic strengths.

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