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Radiation Pattern of a Microstrip Antenna on a Trapezoidal Substrate

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Abstract— This paper deals with a rectangular microstrip antenna on a trapezoidal substrate. It finds radiation pattern of the antenna using the concept of fractional cross product. Results show that as the fraction goes from 1 to 0.1, the direction of null in the H-plane moves from end fire towards broad side. Also, a back-lobe starts to appear in the H-plane.

Keywords— Fractional cross product (FCP), Microstrip antenna, Radiation pattern.

I. Introduction

Popularity of microstrip antenna comes from its advantages like low profile, conformability, low cost and different radiating patch shapes as per requirement. The radiating patch can take any possible shape like rectangle, circle, triangle, ellipse and other regular or irregular geometry. Out of which rectangular patches are the most utilized patch geometry. The comparison and analysis of different shape of patch is done by many researchers [1,2,3] in past. Radiation pattern is perhaps the most important characteristics of an antenna. For microstrip antenna, reported pattern analyses consider the plane intersecting the ground and passing through the radiating edge of the patch or to be perpendicular up to the ground plane. It results in an equivalent rectangular slot between the patch and the ground plane. However to the best of our knowledge there is no reported analysis on radiation pattern of this slot is inclined between the patches and the ground plane. In this paper we consider such inclination of analyzing radiation pattern.

Without inclination the normal to the slot is aligned to the length of the patch. With inclination this normal makes an angle. In other words without inclination the normal is parallel to one of the principal axes and with inclination it makes an angle with principal axes. To obtain an expression for radiation pattern, the slot is modelled as an equivalent magnetic current. Fourier transform of the magnetic current density gives the radiation pattern in the far field. The magnetic current density comes from the cross product of electric field in the slot and the unit vector normal to the slot. Both these vectors are orthogonal to each other where the slot is uninclined. So, the magnetic current is orthogonal to both these vectors. Inclination introduces a perturbation to this situation. Therefore, it becomes difficult to obtain an expression for the radiation pattern. To address this issue we propose to use fractional vector calculus as explained later. Particularly we use fractional cross product between two vectors in our derivation.

The basis of antenna theory is maxwell's equation of electromagnetism. In differential form these equations use divergence and curl operations. These operations are based on differential calculus. Fractional differentiation finds application [4, 5, 6] in various branches of science and technology. Therefore, it is natural to extend this to the domain of electromagnetism. Engheta [7] introduced the concept of fractional calculus in the curl operation and proposed a fractional curl operator. It involved Fourier transform of the curl equation and resulting in vector algebraic equations in the Fourier domain. The curl operation becomes a cross product operation in the Fourier domain. Accordingly, concept of fractional cross product, synonymous to fractional curl, was initiated. Later, these theories found applications in reflection of electromagnetic waves from conducting surfaces [8], propagation of electromagnetic wave in chiral medium [9, 10].

In this work we use fractional cross product. Application of geometrical perspective [11] eliminates fuzziness associated with fractional cross product discussed earlier.

II. Radiation model

The microstrip antenna has two radiating edges and two non-radiating edges. The radiation model considers space between the ground plane and the radiating edge as a slot. The electric field in this slot is responsible for radiation. A slot is equivalent to a magnetic current ($\boldsymbol{M_s}$) obtained from the cross product of normal to the slot ($\hat{n}$) with the electric field ($\boldsymbol{E}$) between the radiating edge and the round plane.

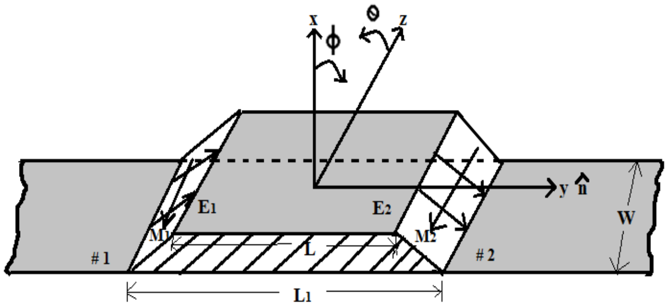

Figure 1. Schematic of a microstrip antenna on a trapezoidal substrate

In this work, the substrate is trapezoidal (figure 1). Therefore, M_s is due to a slot which is at an inclination from the ground plane to the patch. $\boldsymbol{E}$ and M_s are respectively along x and z axes. So, $\hat{n}$ is not orthogonal to the M_s as in case of a simple rectangular patch antenna. M_s comes from the cross product of $\hat{n}$ and $\boldsymbol{E}$ but not orthogonal to $\hat{n}$. Therefore, concept of integral cross product is insufficient in obtaining M_s .

Figure 2 helps in explaining the geometrical concept of fractional cross product. It contains two vectors $\boldsymbol{E}$ and $\hat{n}$ representing electric field and the normal to the slot. A projection of $\hat{n}$ to $\boldsymbol{E}$ is $|\hat{n}|\cos\gamma$. Orthogonal to this projection is $|\hat{n}|\sin\gamma$. Direction of the integral cross product $\hat{n}\times\boldsymbol{E} = |\hat{n}|\,|\boldsymbol{E}|\sin\gamma\,\widehat{M}_s$ is along $\boldsymbol{E}$ rotated anticlockwise about the axis $|\hat{n}|\sin\gamma$ by $\frac{\pi}{2}$ shown as dashed line in the figure. This line is orthogonal to both $\hat{n}$ and $\boldsymbol{E}$. For fractional cross product the rotation of $\boldsymbol{E}$ about the axis $|\hat{n}|\sin\gamma$ is by an amount $\alpha\frac{\pi}{2}$ $(0\le\alpha\le 1)$ shown by solid line. The symbolic representation of fractional cross product is $(\hat{n}\times)^{\alpha}\,\boldsymbol{E}$, where $(\hat{n}\times)^{\alpha}$ is a linear operator. Table 1 shows the result for fractional cross operation on unit vectors along principal coordinate axes.

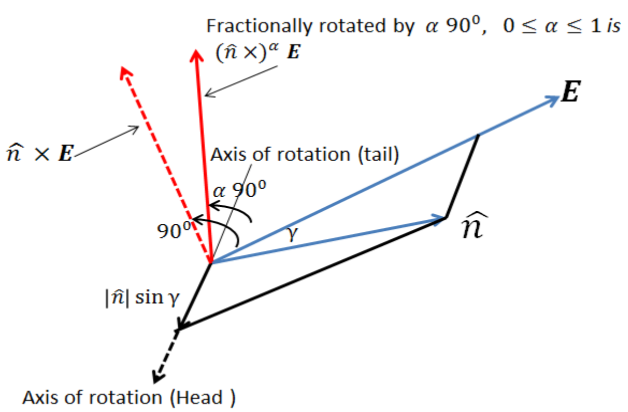

Figure 2. Geometrical concept of fractional cross product

Table 1. Fractional cross products on unit vectors

$(\hat{z}\times)^{\alpha}\hat{x}$	$=\left[\cos\left(\frac{\alpha\pi}{2}\right)\right]\hat{x}+\left[\sin\left(\frac{\alpha\pi}{2}\right)\right]\hat{y}$
$(\hat{z}\times)^{\alpha}\hat{y}$	$=\left[-\sin\left(\frac{\alpha\pi}{2}\right)\right]\hat{x}+\left[\cos\left(\frac{\alpha\pi}{2}\right)\right]\hat{y}$
$(\hat{z}\times)^{\alpha}\hat{z}$	$=0$
$(\hat{x}\times)^{\alpha}\hat{y}$	$=\left[\cos\left(\frac{\alpha\pi}{2}\right)\right]\hat{y}+\left[\sin\left(\frac{\alpha\pi}{2}\right)\right]\hat{z}$
$(\hat{x}\times)^{\alpha}\hat{z}$	$=\left[-\sin\left(\frac{\alpha\pi}{2}\right)\right]\hat{y}+\left[\cos\left(\frac{\alpha\pi}{2}\right)\right]\hat{z}$
$(\hat{y}\times)^{\alpha}\hat{z}$	$=\left[\sin\left(\frac{\alpha\pi}{2}\right)\right]\hat{x}+\left[\cos\left(\frac{\alpha\pi}{2}\right)\right]\hat{z}$
$(\hat{y}\times)^{\alpha}\hat{x}$	$=\left[\cos\left(\frac{\alpha\pi}{2}\right)\right]\hat{x}+\left[-\sin\left(\frac{\alpha\pi}{2}\right)\right]\hat{z}$

III. RADIATION PATTERN OF THE ANTENNA

Standard procedure [12, 13] gives the radiation pattern in the far field. In this procedure, image theory creates image of M_s on the opposite side of the ground plane. This is equivalent to removing the ground plane and doubling M_s. Corresponding to two radiating edges, there are two magnetic current sources separated by a distance L_{eff}. This distance is due to fringing fields given as

$$L_{eff} = L + 2\Delta L \tag{1}$$

where $\Delta L = 0.412\,h\,\frac{\varepsilon_{reff}+0.3}{\varepsilon_{reff}-0.258}\left(\frac{\frac{W}{h}+0.264}{\frac{W}{h}+0.813}\right)$ is the extended length of the patch.

and $\varepsilon_{reff} = \frac{\varepsilon_r+1}{2} + \frac{\varepsilon_r-1}{2}\left(1+\frac{12h}{W}\right)^{-0.5}$ is the effective dielectric constant.

ε_r is dielectric constant of substrate, h is the height of dielectric substrate and W is width of the patch of antenna (figure 1).

Thus it is a two element array of magnetic sources. The array factor (AF) of this array is

$$\mathrm{AF} = 2\cos\left(\frac{k_0 L_{eff}\sin\theta\sin\phi}{2}\right) \tag{2}$$

The element pattern $f(\theta,\phi)$ comes from Fourier transform (FT) of M_s . Since M_s exists only within the slot, so the dimension of the slot determines the limit of integration for finding the FT. For integration it is important to obtain M_s. As discussed already, for the microstrip antenna on trapezoidal substrate, fractional cross operation on **E** gives the M_s.

$$M_s = -2(\hat{n}\times)^{\alpha}\boldsymbol{E} \tag{3}$$

As $\hat{n}$ is along y-axis (for normal microstrip antenna) and $\boldsymbol{E}$ $(=E_0\hat{x})$ is along x-axis, so

$$M_s = -2\,[(\hat{y}\times)^{\alpha}\ \hat{x}]E_0 \quad (4)$$

From table 1,

$$(\hat{y}\times)^{\alpha}\ \hat{x} = \left[\cos\left(\frac{\alpha\pi}{2}\right)\right]\hat{x} + \left[-\sin\left(\frac{\alpha\pi}{2}\right)\right]\hat{z} \quad (5)$$

Therefore,

$$M_s = -2E_0\left\{\left[\cos\left(\frac{\alpha\pi}{2}\right)\right]\hat{x} - \left[\sin\left(\frac{\alpha\pi}{2}\right)\right]\hat{z}\right\} \quad (6)$$

In this expression M_s is independent on x and y. So it can be taken out of integration in the FT and accordingly after conversion to spherical coordinate system patterns for the magnetic current element are,

$$E_\theta = -j\frac{k_0 e^{-jk_0 r}}{4\pi r}N_\phi,\quad N_\phi = 0 \quad (7)$$

So,

$$E_\theta = 0 \quad (8)$$

$$E_\phi = j\frac{k_0 e^{-jk_0 r}}{4\pi r}N_\theta,$$

$$N_\theta = -2E_0 Wh\frac{\sin(Z)}{Z}\frac{\sin(X)}{X}$$

$$\left[\cos\theta\cos\phi\left[\cos\left(\frac{\alpha\pi}{2}\right)\right] + \sin\theta\left[\sin\left(\frac{\alpha\pi}{2}\right)\right]\right] \quad (9)$$

So, $$E_\phi = -j\frac{k_0\,\mathrm{w\,h}\,E_0 e^{-jk_0 r}}{2\pi r}\frac{\sin(Z)}{Z}\frac{\sin(X)}{X}$$

$$\left[\cos\theta\cos\phi\left[\cos\left(\frac{\alpha\pi}{2}\right)\right]\hat{a}_x + \sin\theta\left[\sin\left(\frac{\alpha\pi}{2}\right)\right]\right] \quad (10)$$

Therefore the total pattern for the microstrip antenna is

$$E_\phi^t = E_\phi\,(\mathrm{AF}) = E_\phi\left[2\cos\left(\frac{k_0 L_{eff}\sin\theta\sin\phi}{2}\right)\right]$$

$$E_\phi^t = -j\frac{k_0\,\mathrm{w}\,V_0 e^{-jk_0 r}}{\pi r}\cdot\frac{\sin\left(\frac{k_0 w}{2}\cos\theta\right)}{\frac{k_0 w}{2}\cos\theta}\cdot\frac{\sin\left(\frac{k_0 h}{2}\cos\phi\sin\theta\right)}{\frac{k_0 h}{2}\cos\phi\sin\theta}\cdot$$

$$\cos\left(\frac{k_0 L_{eff}\sin\theta\sin\phi}{2}\right)$$

$$\left[\cos\theta\cos\phi\left[\cos\left(\frac{\alpha\pi}{2}\right)\right] + \sin\theta\left[\sin\left(\frac{\alpha\pi}{2}\right)\right]\right] \quad (11)$$

Where, $hE_0 = V_0$ is the voltage between the patch edge and the ground plane.

Setting condition for E −Plane:

$$\theta = 90^0, 0^0 \le \phi \le 90^0, 270^0 \le \phi \le 360^0$$

equation (11) gives,

$$E_\phi^t\,(E-plane) = -j\frac{k_0\,\mathrm{w}\,V_0 e^{-jk_0 r}}{\pi r}\cdot\frac{\sin\left(\frac{k_0 h}{2}\cos\phi\right)}{\frac{k_0 h}{2}\cos\phi}\cdot$$

$$\cos\left(\frac{k_0 L_{eff}\sin\phi}{2}\right)\left[\sin\left(\frac{\alpha\pi}{2}\right)\right] \quad (12)$$

Setting condition for H −Plane:

$$\phi = 0^0,\ 0^0 \le \theta \le 180^0$$

equation (11) gives,

$$E_\phi^t\,(H-plane) = -j\frac{k_0\,\mathrm{w}\,V_0 e^{-jk_0 r}}{\pi r}\cdot\frac{\sin\left(\frac{k_0 w}{2}\cos\theta\right)}{\frac{k_0 w}{2}\cos\theta}\cdot\frac{\sin\left(\frac{k_0 h}{2}\sin\theta\right)}{\frac{k_0 h}{2}\sin\theta}$$

$$\left[\cos\theta\left[\cos\left(\frac{\alpha\pi}{2}\right)\right]\hat{a}_x + \sin\theta\left[\sin\left(\frac{\alpha\pi}{2}\right)\right]\right] \quad (13)$$

IV. Results and discussions

The present study has considered the following values, dielectric constant (ε_r) = 2.2, height of the dielectric substrate (h) = 0.1588 cm, position of the recessed feed point relative to the leading radiating edge of the patch = 0.3126 cm, length of the patch (L) = 0.9061 cm, width of the patch (W) = 1.186 cm and operating frequency 10 GHz. For simulation, equations (12) and (13) are implemented in MATLAB. Figure 3 shows the radiation pattern for α = 1. Under this condition the radiator is a normal rectangular microstrip antenna. Simulation shows that pattern is omni directional in E-plane and figure of 8 in H-plane. In E and H planes the HPBW (half power band width) are 88^0 and 76^0 respectively. The directivity of antenna is 5.3506 (i.e. 7.2841dB). Figure 4 shows the pattern for α = 0.95. The E-plane pattern remains omni while in the H-plane pattern there is a small but prominent back lobe. The E- and H- plane HPBW are 88^0 and 82^0 respectively. So, decrease of α affects the H-plane HPBW. The directivity of this antenna is 5.3629 (i.e. 7.2940 dB). A small incremental change in directivity is evident. Figure 5 shows the pattern for α = 0.8. The HPBW for E- and H-plane are 80^0 and 102^0 respectively. So, with appreciable decrease in , HPBW increases for H-plane but decreases in E-plane. However, the overall directivity increases to 5.5527 (i.e.7.4450 dB). Figure 6 shows the pattern when α = 0.1. For this sufficient decrease in α, H-plane HPBW shows 180^0 degree. The peaks of E- and H-plane patterns are in opposite directions. The null in the H-plane pattern is in the direction of E-plane pattern peak. The directivity of the antenna increases sufficiently to10.4138 (i.e.10.1761dB).

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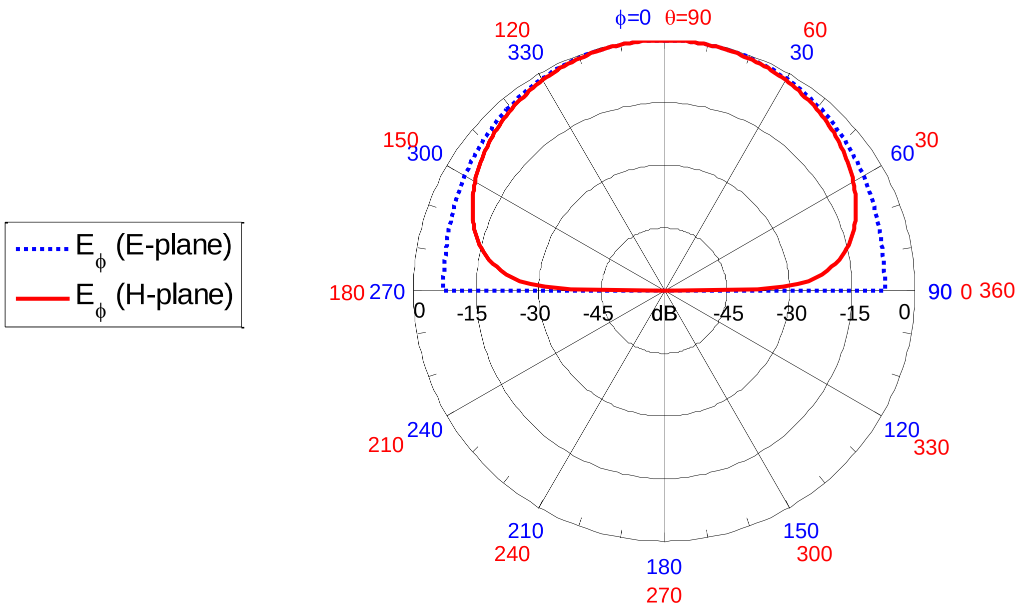

Figure 3. E and H plane pattern of trapezoidal microstrip antenna for $\alpha = 1$

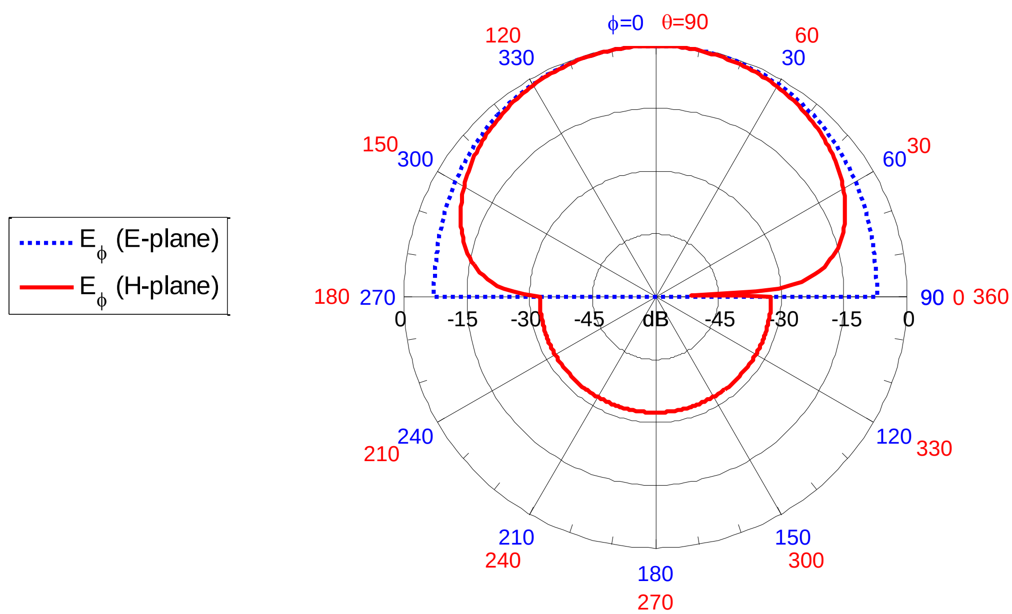

Figure 4. E and H plane pattern of trapezoidal microstrip antenna for $\alpha = 0.95$

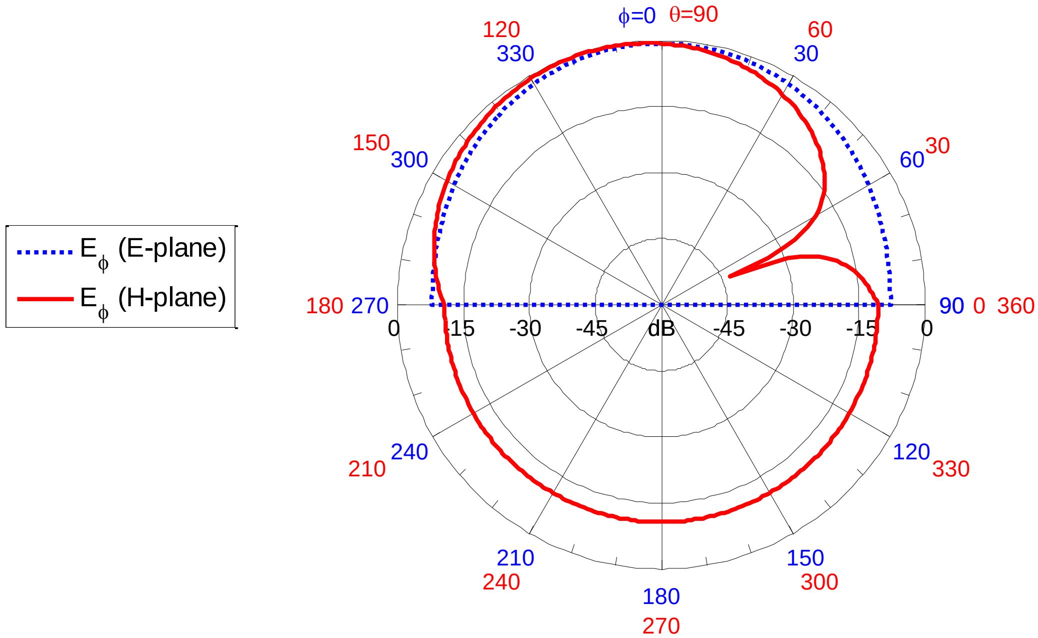

Figure 5. E and H plane pattern of trapezoidal microstrip antenna for α = 0.8

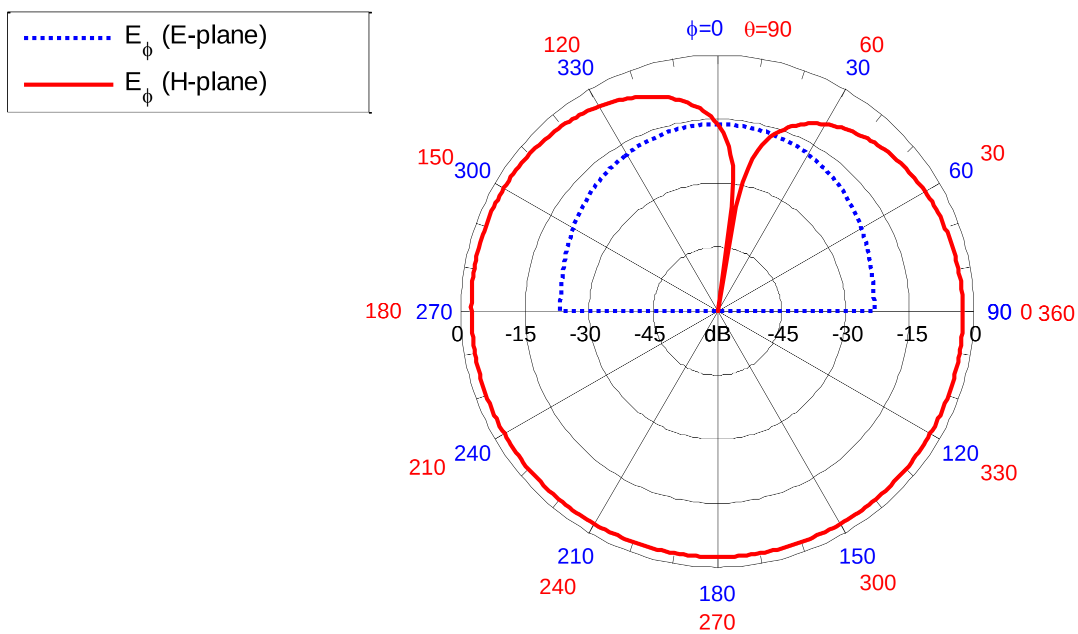

Figure 6. E and H plane pattern of trapezoidal microstrip antenna for α = 0.1

V. Conclusion

A trapezoidal substrate is more directive compare to that on a brick substrate. The appropriate choice of α, the H-plane null can be placed as per requirement. Therefore, a rectangular patch antenna on a trapezoidal surface is advantageous.

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