Neutrosophic Generalized Semi Alpha Star Compact, Connected, Regular and Normal Spaces in Topological Spaces

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***Abstract*—- Real-life structures always include indeterminacy. The Mathematical tool which is well known in dealing with indeterminacy is Neutrosophic. Smarandache proposed the approach of Neutrosophic sets. Neutrosophic sets deal with uncertain data. The notion of Neutrosophic set is generally referred to as the generalization of intuitionistic fuzzy set. In 2021, P. Anbarasi Rodrigo and S. Maheswari introduced new concepts of Neutrosophic closed sets namely Neutrosophic generalized semi alpha star closed** (briefly $N_{eu}gs\alpha^*$-closed) **sets and** $N_{eu}gs\alpha^*$-open **sets as well as** $N_{eu}gs\alpha^*$-continuity **in Neutrosophic topological spaces and studied their some properties. In this chapter, we introduce the notions of** $N_{eu}gs\alpha^*$-compact spaces, $N_{eu}gs\alpha^*$-Lindelof **space, countably** $N_{eu}gs\alpha^*$-compact **spaces,** $N_{eu}gs\alpha^*$-connected **spaces,** $N_{eu}gs\alpha^*$-cseparated **sets,** $N_{eu}-Super-gs\alpha^*$-connected **spaces,** $N_{eu}-Extremely-gs\alpha^*$-disconnected **spaces, and** $N_{eu}-Strongly-gs\alpha^*$-connected **spaces,** $N_{eu}gs\alpha^*$-Regular **spaces, strongly** $N_{eu}gs\alpha^*$-Regular **spaces,** $N_{eu}gs\alpha^*$-Normal **spaces, and strongly** $N_{eu}gs\alpha^*$-Normal **spaces by using** $N_{eu}gs\alpha^*$-open **sets and** $N_{eu}gs\alpha^*$-closed **sets in Neutrosophic topological spaces. We study their basic properties and fundaments characteristics of these spaces in Neurosophic topological spaces.**

Keywords: $N_{eu}gs\alpha^*$-closed **set,** $N_{eu}gs\alpha^*$-open **set,** $N_{eu}gs\alpha^*$-compact **spaces,** $N_{eu}gs\alpha^*$-Lindelof spaces, Countably $N_{eu}gs\alpha^*$-compact spaces, $N_{eu}gs\alpha^*$-connected spaces, $N_{eu}gs\alpha^*$-separated sets, $N_{eu}-Super-gs\alpha^*$-connected spaces, $N_{eu}-Extremely-gs\alpha^*$-disconnected spaces, $N_{eu}-Strongly-gs\alpha^*$-connected spaces, $N_{eu}gs\alpha^*$-Regular spaces, Strongly $N_{eu}gs\alpha^*$-Regular spaces, $N_{eu}gs\alpha^*$-Normal spaces, Strongly $N_{eu}gs\alpha^*$-Normal spaces

I.INTRODUCTION

Many real-life problems in Business, Finance, Medical Sciences, Engineering, and Social Sciences deal with uncertainties. There are difficulties in solving the uncertainties in data by traditional mathematical models. There are approaches such as fuzzy sets, intuitionistic fuzzy sets, vague sets, and rough sets which can be treated as mathematical tools to avert obstacles dealing with ambiguous data. But all these approaches have their implicit crisis in solving the problems involving indeterminant and inconsistent data due to inadequacy of parameterization tools. Molodtsov introduced soft set theory. Smarandache studies neutrosophic set as an approach for solving issues that cover unreliable, indeterminacy and persistent data. Applications of neutrosophic topology depend upon the properties of neutrosophic closed sets, neutrosophic interior operator, and neutrosophic open sets. In 2021, P. Anbarasi Rodrigo and S. Maheswari introduced new concepts of Neutrosophic closed sets namely Neutrosophic generalized semi alpha star closed (briefly $N_{eu}-gs\alpha^*$-closed) sets and $N_{eu}-gs\alpha^*$-open sets as well as $N_{eu}-gs\alpha^*$-continuity in Neutrosophic topological spaces and studied their some properties. In this chapter, we introduce $N_{eu}gs\alpha^*$-compact spaces, $N_{eu}gs\alpha^*$-Lindelof spaces, Countably $N_{eu}gs\alpha*$-compact spaces, $N_{eu}gs\alpha*$-connected spaces, $N_{eu}gs\alpha*$-separated sets, N_{eu}-Super-$gs\alpha*$-connected spaces, N_{eu}-Extremely-$gs\alpha*$-disconnected spaces, N_{eu}-Strongly-$gs\alpha*$-connected spaces, $N_{eu}gs\alpha*$-Regular spaces, strongly $N_{eu}gs\alpha*$-Regular spaces, $N_{eu}gs\alpha*$-Normal space, and strongly $N_{eu}gs\alpha*$-Normal spaces by using neutrosophic $N_{eu}gs\alpha*$-open sets, and $N_{eu}gs\alpha*$-closed sets in neutrosophic topological spaces. We investigate their several basic properties and characterizations in neutrosophic topological spaces.

II. PRELIMINARIES

Definition 2.1. Let X be a non-empty fixed set. A neutrosophic set (briefly N_{eu}-set) P is an object having the form $P=\{\langle x,\mu_P(x),\sigma_P(x),\gamma_P(x)\rangle : x\in X\}$, *where* $\mu_P(x)$- represents the degree of membership, $\sigma_P(x)$- represents the degree of indeterminacy, and $\gamma_P(x)$- represents the degree of non-membership.

Definition 2.2. A neutrosophic topology on a non-empty set X is a family T_N of neutrosophic subsets of X satisfying (i) $0_N, 1_N \in T_N$. (ii) $G\cap H\in T_N$ for every $G, H\in T_N$, (iii) $\bigcup_{j\in J} G_j \in T_N$ for every $\{G_j : j\in J\}\subseteq \tau_N$. Then the pair (X,T_N) is called a neutrosophic topological space (briefly, N_{eu}-Top-Space). The elements of T_N are called neutrosophic open (briefly N_{eu}-open) sets in X. A neutrosophic set A in X is called a neutrosophic closed (briefly N_{eu}-closed) set if and only if its complement A^C is a N_{eu}-open set.

Definition 2.3. Let (X,T_N) be a N_{eu}-Top-Space and A be a N_{eu}-set. Then

(i) The neutrosophic interior of A, denoted by $N_{eu}Int(A)$ is the union of all N_{eu}-open subsets of A.

(ii) The neutrosophic closure of A denoted by $N_{eu}Cl(A)$ is the intersection of all N_{eu}-closed sets containing A.

Definition 2.4. Let A be a N_{eu}-subset of a N_{eu}-Top-Space (X,T_N). Then A is said to be a neutrosophic regular open set if $A\subseteq N_{eu}Int[N_{eu}Cl(A)]$. The complement of a neutrosophic regular open set is called a neutrosophic regular closed set in X.

Definition 2.5. Let (X,T_N) be a N_{eu}-Top-Space and A be a N_{eu}-set of X. Then A is said to be a neutrosophic α-closed (briefly $N_{eu}\alpha$-closed) set if $N_{eu}Cl[N_{eu}Int(N_{eu}Cl(A))]\subseteq A$.

Definition 2.6. Let (X,T_N) be a N_{eu}-Top-Space and A be a N_{eu}-set of X. Then A is said to be a neuromorphic $\alpha*$-open (briefly $N_{eu}\alpha*$-open) set if $A\subseteq N_{eu}\alpha Int[N_{eu}Cl(N_{eu}\alpha Int(A))]$.

Definition 2.7. Let (X,T_N) be a N_{eu}-Top-Space and A be a N_{eu}-set of X. Then A is said to be a neuromorphic generalized semi alpha star closed (briefly $N_{eu}gs\alpha*$-closed) set if $N_{eu}\alpha Int[N_{eu}\alpha Cl(A)]\subseteq N_{eu}Int(G)$ whenever $A\subseteq G$ and G is $N_{eu}\alpha*$-open set.

Theorem 2.8. [Theorem 3.30; Rodrigo et al 2021] In a N_{eu}-Top-Space (X,T_N) we have the following conditions.

(i) $0_{N_{eu}}$ and $1_{N_{eu}}$ are $N_{eu}gs\alpha*$-closed sets in (X,T_N).

(ii) The intersection of any number of $N_{eu}gs\alpha*$-closed sets is $N_{eu}gs\alpha*$-closed set in (X,T_N).

(iii) The union of any two $N_{eu}gs\alpha*$-closed sets is $N_{eu}gs\alpha*$-closed set in (X,T_N).

Definition 2.9. A N_{eu}-set A in a N_{eu}-Top-Space (X,T_N) is called a neutrosophic generalized semi alpha star open (briefly $N_{eu}gs\alpha*$-open) set if $N_{eu}Cl(G)\subseteq N_{eu}\alpha Cl[N_{eu}\alpha Int(A)]$ whenever $G\subseteq A$ and G is $N_{eu}\alpha*$-closed set. A is a $N_{eu}gs\alpha*$-closed set if its complement A^C is a $N_{eu}gs\alpha*$-open set in X. The collection of all $N_{eu}gs\alpha*$-open (resp. $N_{eu}gs\alpha*$-closed) sets in a N_{eu}-Top-Space (X,T_N) is denoted by $N_{eu}gs\alpha*$-$O(X,T_N)$ [resp. $N_{eu}gs\alpha*$-$C(X,T_N)$].

Theorem 2.10. Every N_{eu}-closed set in a N_{eu}-Top-Space (X,T_N) is a $N_{eu}gs\alpha*$-closed set in (X,T_N).

Definition 2.11. A N_{eu}-set A in a N_{eu}-Top-Space (X,T_N) is called a neutrosophic generalized semi alpha star interior of A $\left(briefly\ \mathrm{N}_{eu}gs\alpha*Int(A)\right)$ and neutrosophic generalized semi alpha star closure of $A\left(briefly\ \mathrm{N}_{eu}gs\alpha*Cl(A)\right)$ are defined as follows:

(i) $\mathrm{N}_{eu}gs\alpha*Int(A)=\bigcup\{G : G\ is\ a\ \mathrm{N}_{eu}gs\alpha*-open\ set\ in\ X\ and\ G\subseteq A\}.$

(ii) $\mathrm{N}_{eu}gs\alpha*Cl(A)=\bigcap\{K : K\ is\ a\ \mathrm{N}_{eu}gs\alpha*-closed\ set\ in\ X\ and\ A\subseteq K\}.$

Theorem 2.12. Let (X,T_N) be a N_{eu}–Top–Space. Then for any N_{eu}–subsets A and B of X, we have

(i) $\mathrm{N}_{eu}gs\alpha*Int(A)\subseteq A\subseteq \mathrm{N}_{eu}gs\alpha*Cl(A)$

(ii) A is $\mathrm{N}_{eu}gs\alpha*$–open set in X if and only if $\mathrm{N}_{eu}gs\alpha*Int(A)=A.$

(iii) A is $\mathrm{N}_{eu}gs\alpha*$–closed set in X if and only if $\mathrm{N}_{eu}gs\alpha*Cl(A)=A.$

(iv) $\mathrm{N}_{eu}gs\alpha*Int\left[\mathrm{N}_{eu}gs\alpha*Int(A)\right]=\mathrm{N}_{eu}gs\alpha*Int(A).$

(v) $\mathrm{N}_{eu}gs\alpha*Cl\left[\mathrm{N}_{eu}gs\alpha*Cl(A)\right]=\mathrm{N}_{eu}gs\alpha*Cl(A).$

(vi) If $A\subseteq B,$ then $\mathrm{N}_{eu}gs\alpha*Int(A)\subseteq \mathrm{N}_{eu}gs\alpha*Int(B)$

(vii) If $A\subseteq B,$ then $\mathrm{N}_{eu}gs\alpha*Cl(A)\subseteq \mathrm{N}_{eu}gs\alpha*Cl(B)$

(viii) $\left(\mathrm{N}_{eu}gs\alpha*Cl(A)\right)^c=\mathrm{N}_{eu}gs\alpha*Int\left(A^c\right)$

(iX) $\left(\mathrm{N}_{eu}gs\alpha*Int(A)\right)^c=\mathrm{N}_{eu}gs\alpha*Cl\left(A^c\right)$

(X) $\mathrm{N}_{eu}gs\alpha*Int\left(0_{\mathrm{N}_{eu}}\right)=0_{\mathrm{N}_{eu}},$ $\mathrm{N}_{eu}gs\alpha*Int\left(1_{\mathrm{N}_{eu}}\right)=1_{\mathrm{N}_{eu}}$

(Xi) $\mathrm{N}_{eu}gs\alpha*Cl\left(0_{\mathrm{N}_{eu}}\right)=0_{\mathrm{N}_{eu}},$ $\mathrm{N}_{eu}gs\alpha*Cl\left(1_{\mathrm{N}_{eu}}\right)=1_{\mathrm{N}_{eu}}$

(Xii) $\mathrm{N}_{eu}gs\alpha*Int(A\bigcap B)=\mathrm{N}_{eu}gs\alpha*Int(A)\bigcap \mathrm{N}_{eu}gs\alpha*Int(B)$

(Xiii) $\mathrm{N}_{eu}gs\alpha*Cl(A\bigcup B)=\mathrm{N}_{eu}gs\alpha*Cl(A)\bigcup \mathrm{N}_{eu}gs\alpha*Cl(B)$

(Xiv) $\mathrm{N}_{eu}gs\alpha*Int(A)\bigcup \mathrm{N}_{eu}gs\alpha*Int(B)\subseteq \mathrm{N}_{eu}gs\alpha*Int(A\bigcup B)$

(Xiii) $\mathrm{N}_{eu}gs\alpha*Cl(A\bigcap B)\subseteq \mathrm{N}_{eu}gs\alpha*Cl(A)\bigcap \mathrm{N}_{eu}gs\alpha*Cl(B)$

Theorem 2.13. In a N_{eu}–Top–Space (X,T_N) we have the following conditions.

(i) $0_{\mathrm{N}_{eu}}$ and $1_{\mathrm{N}_{eu}}$ are $\mathrm{N}_{eu}gs\alpha*$–open sets in (X,T_N).

(ii) The union of any number of $\mathrm{N}_{eu}gs\alpha*$–open sets is $\mathrm{N}_{eu}gs\alpha*$–open set in (X,T_N).

(iii) The intersection of any two $\mathrm{N}_{eu}gs\alpha*$–open sets is $\mathrm{N}_{eu}gs\alpha*$–open set in (X,T_N).

Definition 2.14. A function $f:(X,\mathrm{T}_N)\to(Y,\sigma_N)$ is called a $\mathrm{N}_{eu}gs\alpha*$–continuous function if $f^{-1}(B)$ is a $\mathrm{N}_{eu}gs\alpha*$–open (resp. $\mathrm{N}_{eu}gs\alpha*$–closed) set in X, for every N_{eu}–open (resp. N_{eu}–closed) set B in Y.

Definition 2.15. A function $f:(X,\mathrm{T}_N)\to(Y,\sigma_N)$ is called a $\mathrm{N}_{eu}gs\alpha*$–irresolute function if $f^{-1}(B)$ is a $\mathrm{N}_{eu}gs\alpha*$–open (resp. $\mathrm{N}_{eu}gs\alpha*$–closed) set in X, for every $\mathrm{N}_{eu}gs\alpha*$–open (resp. $\mathrm{N}_{eu}gs\alpha*$–closed) set B in Y.

III. NEUTROSOPHIC GENERALIZED SEMI ALPHA STAR COMPACT SPACES

In this section, we introduce $\mathrm{N}_{eu}gs\alpha*$–compact space, $\mathrm{N}_{eu}gs\alpha*$–Lindelof space, and countably $\mathrm{N}_{eu}gs\alpha*$–compact space and investigate their basic properties and characterizations.

Definition 3.1. A collection $\{A_i : i\in I\}$ of N_{eu}–open (resp. $\mathrm{N}_{eu}gs\alpha*$–open) sets in a N_{eu}–Top–Space (X,T_N) is called a N_{eu}–open (resp. $\mathrm{N}_{eu}gs\alpha*$–open) cover of a subset B of X if $B\subseteq\bigcup\{A_i : i\in I\}$ holds.

Definition 3.2. A subset B of a N_{eu}–Top–Space (X,T_N) is said to be N_{eu}–compact (resp. $\mathrm{N}_{eu}gs\alpha*$–compact) relative to (X,T_N), if for every collections $\{A_i : i\in I\}$ of N_{eu}–open (resp. $\mathrm{N}_{eu}gs\alpha*$–open) subsets of (X,T_N) such that $B\subseteq\bigcup\{A_i : i\in I\}$ there exists a finite subset I_0 of I such that $B\subseteq\bigcup\{A_i : i\in I_0\}$.

Definition 3.3. A subset B of a N_{eu}–Top–Space (X,T_N) is called N_{eu}–compact (resp $\mathrm{N}_{eu}gs\alpha*$–compact) if B is

N_{eu}-compact (resp. $N_{eu}gs\alpha^*$-compact) as a subspace of X.

Theorem 3.4. A $N_{eu}gs\alpha^*$-closed subset of a $N_{eu}gs\alpha^*$-compact (X, T_N) is $N_{eu}gs\alpha^*$-compact relative to (X, T_N).

Proof. Let A be a $N_{eu}gs\alpha^*$-closed subset of a $N_{eu}gs\alpha^*$-compact space (X, T_N). Then A^C is $N_{eu}gs\alpha^*$-open in (X, T_N). Let $S = \{A_i : i \in I\}$ be a $N_{eu}gs\alpha^*$-open cover of A by $N_{eu}gs\alpha^*$-open subsets of (X, T_N). Then $S^* = S \bigcup \{A^C\}$ is a $N_{eu}gs\alpha^*$-open cover of (X, T_N). That is $X = \left(\bigcup_{i\in I} A_i\right) \bigcup A^C$. By hypothesis (X, T_N) is $N_{eu}gs\alpha^*$-compact and hence S^* is reducible to a finite subcover of (X, T_N) say $X = A_{i_1} \bigcup A_{i_2} \ldots\ldots \bigcup A_{i_n} \bigcup A^C$, $A_{i_k} \in S \subseteq S^*$. Then $A = A_{i_1} \bigcup A_{i_2} \ldots\ldots \bigcup A_{i_n}$. Thus a $N_{eu}gs\alpha^*$-open cover S of A contains a finite subcover. Hence A is $N_{eu}gs\alpha^*$-compact relative to (X, T_N).

Theorem 3.5. A N_{eu}-Top-Space (X, T_N) is $N_{eu}gs\alpha^*$-compact if and only if every family of $N_{eu}gs\alpha^*$-closed sets of (X, T_N) having finite intersection property has a non-empty intersection.

Proof. Suppose (X, T_N) is $N_{eu}gs\alpha^*$-compact. Let $\{A_i : i \in I\}$ be a family of $N_{eu}gs\alpha^*$-closed sets with finite intersection property. Suppose $\bigcap_{i\in I} A_i = \phi$. Then $X - \bigcap_{i\in I} A_i = X$. This implies $\bigcup_{i\in I}(X - A_i) = X$. Thus the cover $\{X - A_i : i \in I\}$ is a $N_{eu}gs\alpha^*$-open cover of (X, T_N). Then the $N_{eu}gs\alpha^*$-open cover $\{X - A_i : i \in I\}$ has a finite subcover say $\{X - A_i : i \in I_0\}$ for some finite subset I_0 of I. This implies $X = \bigcup_{i\in I_0}(X - A_i)$, which implies $X - \bigcup_{i\in I_0}(X - A_i) = \phi$ which implies $\bigcap_{i\in I_0} A_i = \phi$. This contradicts the assumption. Hence $\bigcap_{i\in I} A_i \neq \phi$.

Conversely, suppose (X, T_N) is not $N_{eu}gs\alpha^*$-compact. Then there exists a $N_{eu}gs\alpha^*$-open cover of (X, T_N) say $\{G_i : i \in I\}$ having no finite subcover. This implies for any finite subfamily $\{G_i : i = 1, 2, ..., n\}$ of $\{G_i : i \in I\}$, we have $\bigcup_{i=1}^{n} G_i \neq X$, which implies $X - \bigcup_{i=1}^{n} G_i \neq X - X$, which implies $\bigcap_{i=1}^{n}(X - G_i) \neq \phi$. Then the family $\{X - G_i : i \in I\}$ of $N_{eu}gs\alpha^*$-closed sets has a finite intersection property. Also, by assumption $\bigcap_{i\in I}(X - G_i) \neq \phi$ which implies $X - \bigcup_{i\in I} G_i \neq \phi$ so that $\bigcup_{i\in I} G_i \neq X$. This implies $\{G_i : i \in I\}$ is not a cover for (X, T_N). This contradicts the fact $\{G_i : i \in I\}$ is a cover for (X, T_N). Therefore a $N_{eu}gs\alpha^*$-open cover $\{G_i : i \in I\}$ of (X, T_N) has a finite subcover $\{G_i : i = 1, 2, ..., n\}$. Hence (X, T_N) is a $N_{eu}gs\alpha^*$-compact.

Theorem 3.6. The image of a $N_{eu}gs\alpha^*$-compact space under a $N_{eu}gs\alpha^*$-irresolute mapping is $N_{eu}gs\alpha^*$-compact.

Proof. Let $f : (X, T_N) \to (Y, \sigma_N)$ be a $N_{eu}gs\alpha^*$-irresolute mapping from a $N_{eu}gs\alpha^*$-compact space (X, T_N) onto a $N_{eu}gs\alpha^*$-Top-Space (Y, σ_N). Let $\{A_i : i \in I\}$ be a $N_{eu}gs\alpha^*$-open cover of (Y, σ_N). Then $\{f^{-1}(A_i) : i \in I\}$ is a $N_{eu}gs\alpha^*$-open cover of (X, T_N), since f is $N_{eu}gs\alpha^*$-irresolute. As (X, T_N) is $N_{eu}gs\alpha^*$-compact, the $N_{eu}gs\alpha^*$-open cover $\{f^{-1}(A_i) : i \in I\}$ of (X, T_N) has a finite subcover $\{f^{-1}(A_i) : i = 1, 2, ..., n\}$. Therefore $X = \bigcup_{i=1}^{n} f^{-1}(A_i)$. Then $f(X) = \bigcup_{i=1}^{n} A_i$, that is $Y = \bigcup_{i=1}^{n} A_i$. Then $\{A_1, A_2, ..., A_n\}$ is a finite subcover of $\{A_i : i \in I\}$ for (Y, σ_N). Hence Y is a $N_{eu}gs\alpha^*$-compact space.

Definition 3.7. A N_{eu}-Top-Space (X, T_N) is countably $N_{eu}gs\alpha^*$-compact if every countable $N_{eu}gs\alpha^*$-open cover of (X, T_N) has a finite subcover.

Definition 3.8. A N_{eu}-Top-Space (X, T_N) is said to be $N_{eu}gs\alpha^*$-Hausdorff if whenever $x_{(\alpha,\beta,\gamma)}$ and $y_{(r,s,t)}$ are distinct points of (X, T_N), there exist disjoint $N_{eu}gs\alpha^*$-open sets A and B of X such that $x_{(\alpha,\beta,\gamma)} \in A$ and $y_{(r,s,t)} \in B$.

Theorem 3.9. Let (X, T_N) be a N_{eu}-Top-Space and (Y, σ_N) be a $N_{eu}gs\alpha^*$-Hausdorff space. If

$f:(X,T_N)\to(Y,\sigma_N)$ is $N_{eu}gs\alpha^*$-irresolute injective mapping, then (X,T_N) is $N_{eu}gs\alpha^*$-Hausdorff.

Proof. Let $x_{(\alpha,\beta,\gamma)}$ and $y_{(r,s,t)}$ be any two distinct neutrosophic points of (X,T_N). Then $f(x_{(\alpha,\beta,\gamma)})$ and $f(y_{(r,s,t)})$ are distinct neutrosophic points of (Y,σ_N), because f is injective. Since (Y,σ_N) is $N_{eu}gs\alpha^*$-Hausdorff, there are disjoint $N_{eu}gs\alpha^*$-open sets G and H in (Y,σ_N) containing $f(x_{(\alpha,\beta,\gamma)})$ and $f(y_{(r,s,t)})$ respectively. Since f is $N_{eu}gs\alpha^*$-irresolute and $G\cap H=\phi$, we have $f^{-1}(G)$ and $f^{-1}(H)$ are disjoint $N_{eu}gs\alpha^*$-open sets in (X,T_N) such that $x_{(\alpha,\beta,\gamma)}\in f^{-1}(G)$ and $y_{(r,s,t)}\in f^{-1}(H)$. Hence (X,T_N) is $N_{eu}gs\alpha^*$-Hausdorff.

Theorem 3.10. If $f:(X,T_N)\to(Y,\sigma_N)$ is $N_{eu}gs\alpha^*$-irresolute and bijective and if X is $N_{eu}gs\alpha^*$-compact and Y is $N_{eu}gs\alpha^*$-Hausdorff, then f is a $N_{eu}gs\alpha^*$-homeomorphism.

Proof. We have to show that the inverse function g of f is $N_{eu}gs\alpha^*$-irresolute. For this we show that if A is $N_{eu}gs\alpha^*$-open in (X,T_N) then the pre-image $g^{-1}(A)$ is $N_{eu}gs\alpha^*$-open in (Y,σ_N). Since the $N_{eu}gs\alpha^*$-open (or $N_{eu}gs\alpha^*$-closed) sets are just the complements of $N_{eu}gs\alpha^*$-closed (resp. $N_{eu}gs\alpha^*$-open) subsets, and $g^{-1}(X-A)=Y-g^{-1}(A)$. We see that the $N_{eu}gs\alpha^*$-irresolute mapping of g is equivalent to: if B is $N_{eu}gs\alpha^*$-closed in (X,T_N) then the pre-image $g^{-1}(B)$ is $N_{eu}gs\alpha^*$-closed in Y. To prove this, let B be a $N_{eu}gs\alpha^*$-closed subset of X. Since g is the inverse of f, we have $g^{-1}(B)=f(B)$, hence we have to show that $f(B)$ is a $N_{eu}gs\alpha^*$-closed set in Y. By theorem 3.4, B is $N_{eu}gs\alpha^*$-compact. By Theorem 3.6, implies that $f(B)$ is $N_{eu}gs\alpha^*$-copmpact. Since Y is $N_{eu}gs\alpha^*$-Hausdorff space implies that $f(B)$ is $N_{eu}gs\alpha^*$-closed in (Y,σ_N).

Definition 3.11. A N_{eu}-Top-Space (X,T_N) is said to be $N_{eu}gs\alpha^*$-Lindelof space if every $N_{eu}gs\alpha^*$-open cover of (X,T_N) has a countable subcover.

Theorem 3.12. Every $N_{eu}gs\alpha^*$-compact space is a $N_{eu}gs\alpha^*$-Lindelof space.

Proof. Let (X,T_N) be $N_{eu}gs\alpha^*$-compact. Let $\{A_i : i\in I\}$ be a $N_{eu}gs\alpha^*$-open cover of (X,T_N). Then $\{A_i : i\in I\}$ has a finite subcover $\{A_i : i=1,2,...,n\}$, since (X,T_N) is neutrosophic $N_{eu}gs\alpha^*$-copmpact. Since every finite subcover is always a countable subcover and therefore, $\{A_i : i=1.2,...,n\}$ is countable subcover of $\{A_i : i\in I\}$ for (X,T_N). Hence (X,T_N) is $N_{eu}gs\alpha^*$-Lindelof space.

Theorem 3.13. The image of a $N_{eu}gs\alpha^*$-Lindelof space under a $N_{eu}gs\alpha^*$-irresolute mapping is $N_{eu}gs\alpha^*$-Lindelof.

Proof. Let $f:(X,T_N)\to(Y,\sigma_N)$ be a $N_{eu}gs\alpha^*$-irresolute mapping from a $N_{eu}gs\alpha^*$-Lindelof space (X,T_N) onto a N_{eu}-Top-Space (Y,σ_N). Let $\{A_i : i\in I\}$ be a neutrosophic $N_{eu}gs\alpha^*$-open cover of (Y,σ_N). Then $\{f^{-1}(A_i) : i\in I\}$ is a $N_{eu}gs\alpha^*$-open cover of (X,T_N), since f is $N_{eu}gs\alpha^*$-irresolute. As (X,T_N) is $N_{eu}gs\alpha^*$-Lindelof, the $N_{eu}gs\alpha^*$-open cover $\{f^{-1}(A_i) : i\in I\}$ of (X,T_N) has a countable subcover $\{f^{-1}(A_i) : i\in I_0\}$ for some countable subset I_0 of I. Therefore $X=\bigcup_{i\in I_0} f^{-1}(A_i)$ which implies $f(X)=Y=\bigcup_{i\in I_0} A_i$, that is $\{A_i : i\in I_0\}$ is a countable subcover of $\{A_i : i\in I\}$ for (Y,σ_N). Hence (Y,σ_N) is $N_{eu}gs\alpha^*$-Lindelof space.

Theorem 3.14. Let (X,T_N) be $N_{eu}gs\alpha^*$-Lindelof and countably $N_{eu}gs\alpha^*$-copmpact space. Then (X,T_N) is $N_{eu}gs\alpha^*$-copmpact.

Proof. Let $\{A_i : i\in I\}$ be a $N_{eu}gs\alpha^*$-open cover of (X,T_N). Since (X,T_N) is $N_{eu}gs\alpha^*$-Lindelof space. Hence $\{A_i : i\in I\}$ has a countable subcover $\{A_{i_n} : n\in N\}$. Therefore, $\{A_{i_n} : n\in N\}$ is a countable subcover of (X,T_N) and $\{A_{i_n} : n\in N\}$ is a subfamily of $\{A_i : i\in I\}$ and so $\{A_{i_n} : n\in N\}$ is a countable

$N_{eu}gs\alpha*$-open cover of (X, T_N). Again since (X, T_N) is countably $N_{eu}gs\alpha*$-copmpact, $\{A_{i_n} : n \in N\}$ has a finite subcover $\{A_{i_k} : k = 1, 2, ..., n\}$. Therefore $\{A_{i_k} : k = 1, 2, ..., n\}$ is a finite subcover of $\{A_i : i \in I\}$ for (X, T_N). Hence (X, T_N) is $N_{eu}gs\alpha*$-copmpact space.

Theorem 3.15. A N_{eu}-Top-Space (X, T_N) is $N_{eu}gs\alpha*$-copmpact if and only if every basic $N_{eu}gs\alpha*$-open cover of (X, T_N) has a finite subcover.

Proof. Let (X, T_N) be $N_{eu}gs\alpha*$-copmpact. Then every $N_{eu}gs\alpha*$-open cover of (X, T_N) has a finite subcover. Conversely, suppose that every basic $N_{eu}gs\alpha*$-open cover of (X, T_N) has a finite subcover and let $C = \{G_\lambda : \lambda \in \Lambda\}$ be any $N_{eu}gs\alpha*$-open cover of (X, T_N). If $B = \{D_\alpha : \alpha \in \Delta\}$ is any $N_{eu}gs\alpha*$-open base for (X, T_N), then each G_λ is union of some members of B and the totality of all such members of B evidently a basic $N_{eu}gs\alpha*$-open cover of (X, T_N). By hypothesis this collection of members of B has a finite subcover, $\{D_{\alpha_i} : i = 1, 2, ..., n\}$. For each D_{α_i} in this finite subcover, we can select a G_{λ_i} from C such that $D_{\alpha_i} \subseteq G_{\lambda_i}$. It follows that the finite subcollection $\{G_{\lambda_i} : i = 1, 2, ..., n\}$, which arises in this way is a subcover of C. Hence (X, T_N) is $N_{eu}gs\alpha*$-copmpact.

IV. NEUTROSOPHIC GENERALIZED SEMI ALPHA STAR CONNECTED SPACES

In this section we introduce and study the notions of $N_{eu}gs\alpha*$-connected spaces, $N_{eu}gs\alpha*$-separated sets, N_{eu}-Super-$gs\alpha*$-connected spaces, N_{eu}-Extremely-$gs\alpha*$-disconnected spaces and N_{eu}-Strongly-$gs\alpha*$-connected spaces in N_{eu}-Top-Spaces.

Definition 4.1. A N_{eu}-Top-Space (X, T_N) is $N_{eu}gs\alpha*$-disconnected if there exist $N_{eu}gs\alpha*$-open sets A, B in X, $A \neq 0_N$, $B \neq 0_N$ such that $A \cup B = 1_N$ and $A \cap B = 0_N$. If (X, T_N) is not $N_{eu}gs\alpha*$-disconnected then it is said to be $N_{eu}gs\alpha*$-connected.

Theorem 4.2. A N_{eu}-Top-Space (X, T_N) is $N_{eu}gs\alpha*$-connected space if and only if there exists no nonempty $N_{eu}gs\alpha*$-open sets U and V in (X, T_N) such that $U = V^C$.

Proof. Necessity: Let U and V be two $N_{eu}gs\alpha*$-open sets in (X, T_N) such that $U \neq 0_N$, $V \neq 0_N$ and $U = V^C$. Therefore V^C is a $N_{eu}gs\alpha*$-closed set. Since $U \neq 0_N$, $V \neq 1_N$. This implies V is a proper N_{eu}-subset which is both $N_{eu}gs\alpha*$-open set and $N_{eu}gs\alpha*$-closed set in X. Hence X is not a $N_{eu}gs\alpha*$-connected space. But this is a contradiction to our hypothesis. Thus, there exist no nonempty $N_{eu}gs\alpha*$-open sets U and V in X, such that $U = V^C$.

Sufficiency: Let U be both $N_{eu}gs\alpha*$-open and $N_{eu}gs\alpha*$-closed set of X such that $U \neq 0_N$, $U \neq 1_N$. Now let $V = U^C$. Then V is a $N_{eu}gs\alpha*$-open set and $V \neq 1_N$. This implies $U^C = V \neq 0_N$, which is a contradiction to our hypothesis. Therefore X is $N_{eu}gs\alpha*$-connected space.

Theorem 4.3. A N_{eu}-Top-Space (X, T_N) is $N_{eu}gs\alpha*$-connected space if and only if there do not exist nonempty N_{eu}-subsets U and V in X such that $U = V^C$, $V = \left[N_{eu}gs\alpha*Cl(U)\right]^C$ and $U = \left[N_{eu}gs\alpha*Cl(V)\right]^C$.

Proof. Necessity: Let U and V be two N_{eu}-subsets of (X, T_N) such that $U \neq 0_N$, $V \neq 0_N$ and $U = V^C$, $V = \left[N_{eu}gs\alpha*Cl(U)\right]^C$ and $U = \left[N_{eu}gs\alpha*Cl(V)\right]^C$. Since $\left[N_{eu}gs\alpha*Cl(U)\right]^C$ and $\left[N_{eu}gs\alpha*Cl(V)\right]^C$ are $N_{eu}gs\alpha*$-open sets in X, so U and V are $N_{eu}gs\alpha*$-open sets in X. This implies X is not a $N_{eu}gs\alpha*$-connected space, which is a contradiction. Therefore, there exist no nonempty $N_{eu}gs\alpha*$-open sets U and V in X, such that $U = V^C$, $V = \left[N_{eu}gs\alpha*Cl(U)\right]^C$ and $U = \left[N_{eu}gs\alpha*Cl(V)\right]^C$.

Sufficiency: Let U be both $N_{eu}gs\alpha*$-open and $N_{eu}gs\alpha*$-closed set in X such that $U \neq 0_N$, $U \neq 1_N$. Now by taking $V = U^C$ we obtain a contradiction to our hypothesis. Hence X is $N_{eu}gs\alpha*$-connected space.

Theorem 4.4. Let $f:(X,T_N)\to(Y,\sigma_N)$ be a $N_{eu}gs\alpha*$-irresolure surjection and X be $N_{eu}gs\alpha*$-connected. Then Y is $N_{eu}gs\alpha*$-connected.

Proof. Assume that Y is not $N_{eu}gs\alpha*$-connected, then there exist nonempty $N_{eu}gs\alpha*$-open sets U and V in Y such that $U\bigcup V=1_N$ and $U\bigcap V=0_N$. Since f is $N_{eu}gs\alpha*$-irresolure mapping, $A=f^{-1}(U)\neq 0_N$, $B=f^{-1}(V)\neq 0_N$, which are $N_{eu}gs\alpha*$-open sets in X and $f^{-1}(U)\bigcup f^{-1}(V)=f^{-1}(1_N)=1_N$, which implies $A\bigcup B=1_N$. Also $f^{-1}(U)\bigcap f^{-1}(V)=f^{-1}(0_N)=0_N$, which implies $A\bigcap B=0_N$. Thus, X is $N_{eu}gs\alpha*$-disconnected, which is a contradiction to our hypothesis. Hence Y is $N_{eu}gs\alpha*$-connected.

Definition 4.5. Let A and B be nonempty N_{eu}-subsets in a N_{eu}-Top-Space (X,T_N). Then A and B are said to be $N_{eu}gs\alpha*$-separated if $N_{eu}gs\alpha*Cl(A)\bigcap B=A\bigcap N_{eu}gs\alpha*Cl(B)=0_N$.

Remark 4.6. Any two disjoint non-empty $N_{eu}gs\alpha*$-closed sets are $N_{eu}gs\alpha*$-separated.

Proof. Suppose A and B are disjoint non-empty $N_{eu}gs\alpha*$-closed sets. Then $N_{eu}gs\alpha*Cl(A)\bigcap B=A\bigcap N_{eu}gs\alpha*Cl(B)=A\bigcap B=0_N$. This shows that A and B are $N_{eu}gs\alpha*$-separated.

Theorem 4.7. (i) Let A and B be two $N_{eu}gs\alpha*$-separated subsets of a N_{eu}-Top-Space (X,T_N) and $C\subseteq A$, $D\subseteq B$. Then C and D are also $N_{eu}gs\alpha*$-separated.

(ii) Let A and B be both $N_{eu}gs\alpha*$-separated subsets of a N_{eu}-Top-Space (X,T_N) and let $H=A\bigcap B^C$ and $G=B\bigcap A^C$. Then H and G are also $N_{eu}gs\alpha*$-separated.

Proof. (i) Let A and B be two $N_{eu}gs\alpha*$-separated sets in N_{eu}-Top-Space (X,T_N). Then $N_{eu}gs\alpha*Cl(A)\bigcap B=0_N=A\bigcap N_{eu}gs\alpha*Cl(B)$. Since $C\subseteq A$ and $D\subseteq B$, then $N_{eu}gs\alpha*Cl(C)\subseteq N_{eu}gs\alpha*Cl(A)$ and $N_{eu}gs\alpha*Cl(D)\subseteq N_{eu}gs\alpha*Cl(B)$. This implies that, $N_{eu}gs\alpha*Cl(C)\bigcap D\subseteq N_{eu}gs\alpha*Cl(A)\bigcap B=0_N$ and hence $N_{eu}gs\alpha*Cl(C)\bigcap D=0_N$. Similarly $N_{eu}gs\alpha*Cl(D)\bigcap C\subseteq N_{eu}gs\alpha*Cl(B)\bigcap A=0_N$ and hence $N_{eu}gs\alpha*Cl(D)\bigcap C=0_N$. Therefore C and D are $N_{eu}gs\alpha*$-separated.

(ii) Let A and B be both $N_{eu}gs\alpha*$-open subsets of X. Then A^C and B^C are $N_{eu}gs\alpha*$-closed sets. Since $H\subseteq B^C$, then $N_{eu}gs\alpha*Cl(H)\subseteq N_{eu}gs\alpha*Cl(B^C)=B^C$ and so $N_{eu}gs\alpha*Cl(H)\bigcap B=0_N$. Since $G\subseteq B$, then $N_{eu}gs\alpha*Cl(H)\bigcap G\subseteq N_{eu}gs\alpha*Cl(H)\bigcap B=0_N$. Thus, we have $N_{eu}gs\alpha*Cl(H)\bigcap G=0_N$. Similarly, $N_{eu}gs\alpha*Cl(G)\bigcap H=0_N$. Hence H and G are $N_{eu}gs\alpha*$-separated.

Theorem 4.8. Two N_{eu}-subsets A and B of a N_{eu}-Top-Space (X,T_N) are $N_{eu}gs\alpha*$-separated if and only if there exist $N_{eu}gs\alpha*$-open sets U and V in X such that $A\subseteq U$, $B\subseteq V$ and $A\bigcap V=0_N$ and $B\bigcap U=0_N$.

Proof. Let A and B be $N_{eu}gs\alpha*$-separated. Then $A\bigcap N_{eu}gs\alpha*Cl(B)=0_N=B\bigcap N_{eu}gs\alpha*Cl(A)$. Take $V=(N_{eu}gs\alpha*Cl(A))^C$ and $U=(N_{eu}gs\alpha*Cl(B))^C$. Then U and V are $N_{eu}gs\alpha*$-open sets in X such that $A\subseteq U$, $B\subseteq V$ and $A\bigcap V=0_N$ and $B\bigcap U=0_N$.

Conversely let U and V be $N_{eu}gs\alpha*$-open sets such that $A\subseteq U$, $B\subseteq V$ and $A\bigcap V=0_N$, $B\bigcap U=0_N$. Then $A\subseteq V^C$ and $B\subseteq U^C$ and V^C and U^C are $N_{eu}gs\alpha*$-closed. This implies $N_{eu}gs\alpha*Cl(A)\subseteq N_{eu}gs\alpha*Cl(V^C)=V^C\subseteq B^C$ and $N_{eu}gs\alpha*Cl(B)\subseteq N_{eu}gs\alpha*Cl(U^C)=U^C\subseteq A^C$. That is, $N_{eu}gs\alpha*Cl(A)\subseteq B^C$ and $N_{eu}gs\alpha*Cl(B)\subseteq A^C$. Therefore $A\bigcap N_{eu}gs\alpha*Cl(B)=0_N=N_{eu}gs\alpha*Cl(A)\bigcap B$. Hence A and B are $N_{eu}gs\alpha*$-separated.

Theorem 4.9. Each two $N_{eu}gs\alpha*$-separated sets are always disjoint.

Proof. Let A and B be $N_{eu}gs\alpha*$-separated. Then $A\bigcap N_{eu}gs\alpha*Cl(B)=0_N=N_{eu}gs\alpha*Cl(A)\bigcap B$. Now, $A\bigcap B\subseteq A\bigcap N_{eu}gs\alpha*Cl(B)=0_N$. Therefore $A\bigcap B=0_N$ and hence A and B are disjoint.

Theorem 4.10. A N_{eu}-Top-Space (X,T_N) is $N_{eu}gs\alpha*$-connected if and only if $A\bigcup B\neq 1_N$, where A and B are $N_{eu}gs\alpha*$-separated sets.

Proof. Assume that (X, T_N) is $N_{eu}gs\alpha*$-connected space. Suppose $A \bigcup B = 1_N$, where A and B are $N_{eu}gs\alpha*$-separated sets. Then $N_{eu}gs\alpha*Cl(A) \bigcap B = A \bigcap N_{eu}gs\alpha*Cl(B) = 0_N$. Since $A \subseteq N_{eu}gs\alpha*Cl(A)$, we have $A \bigcap B \subseteq N_{eu}gs\alpha*Cl(A) \bigcap B = 0_N$. Therefore $N_{eu}gs\alpha*Cl(A) \subseteq B^C = A$ and $N_{eu}gs\alpha*Cl(B) \subseteq A^C = B$. Hence $A = N_{eu}gs\alpha*Cl(A)$ and $B = N_{eu}gs\alpha*Cl(B)$. Therefore A and B are $N_{eu}gs\alpha*$-closed sets and hence $A = B^C$ and $B = A^C$ are disjoint $N_{eu}gs\alpha*$-open sets. Thus $A \neq 0_N$, $B \neq 0_N$ such that $A \bigcup B = 1_N$ and $A \bigcap B = 0_N$, A and B are $N_{eu}gs\alpha*$-open sets. That is X is not $N_{eu}gs\alpha*$-connected, which is a contradiction to X is a $N_{eu}gs\alpha*$-connected space. Hence 1_N is not the union of any two $N_{eu}gs\alpha*$-separated sets.

Conversely, assume that 1_N is not the union of any two $N_{eu}gs\alpha*$-separated sets. Suppose X is not $N_{eu}gs\alpha*$-connected. Then $A \bigcup B = 1_N$, where $A \neq 0_N$, $B \neq 0_N$ such that $A \bigcap B = 1_N$, A and B are $N_{eu}gs\alpha*$-open sets in X. Since $A \subseteq B^C$ and $B \subseteq A^C$, $N_{eu}gs\alpha*Cl(A) \bigcap B \subseteq B^C \bigcap B = 0_N$ and $A \bigcap N_{eu}gs\alpha*Cl(B) \subseteq A \bigcap A^C = 0_N$. That is A and B are $N_{eu}gs\alpha*$-separated sets. This is a contradiction. Therefore X is $N_{eu}gs\alpha*$-connected.

Definition 4.11. A N_{eu}-Top-Space (X, T_N) is N_{eu}-Super-$gs\alpha*$-disconnected if there exists a -$gs\alpha*$-N_{eu}-Regular-$gs\alpha*$-open set A in X such that $A \neq 0_N$ and $A \neq 1_N$. A N_{eu}-Top-Spaces (X, T_N) is called N_{eu}-Super-$gs\alpha*$-connected if X is not N_{eu}-Super-$gs\alpha*$-disconnected.

Theorem 4.12. Let (X, T_N) be a N_{eu}-Top-Space. Then following assertions are equivalent:

(i) X is N_{eu}-Super-$gs\alpha*$-connected.

(ii) For each $N_{eu}gs\alpha*$-open set $U \neq 0_N$ in X, we have $N_{eu}gs\alpha*Cl(U) = 1_N$.

(iii) For each $N_{eu}gs\alpha*$-closed set $U \neq 1_N$ in X, we have $N_{eu}gs\alpha*Int(U) = 0_N$.

(iv) There do not exist $N_{eu}gs\alpha*$-open subsets U and V in (X, T_N), such that $U \neq 0_N$, $V \neq 0_N$ and $U \subseteq V^C$.

(v) There do not exist $N_{eu}gs\alpha*$-open subsets U and V in (X, T_N), such that $U \neq 0_N$, $V \neq 0_N$, $V = \left(N_{eu}gs\alpha*Cl(U)\right)^C$ and $U = \left(N_{eu}gs\alpha*Cl(V)\right)^C$.

(vi) There do not exist $N_{eu}gs\alpha*$-closed subsets U and V in (X, T_N), such that $U \neq 1_N$, $V \neq 1_N$, $V = \left(N_{eu}gs\alpha*Int(U)\right)^C$ and $U = \left(N_{eu}gs\alpha*Int(V)\right)^C$.

Proof. (i)$\Rightarrow$(ii): Assume that there exists a $N_{eu}gs\alpha*$-open set $A \neq 0_N$ such that $N_{eu}gs\alpha*Cl(A) \neq 1_N$. Now take $B = N_{eu}gs\alpha*Int\left[N_{eu}gs\alpha*Cl(A)\right]$. Then B is a proper N_{eu}-Regular-$gs\alpha*$-open set in X which contradicts that X is N_{eu}-Super-$gs\alpha*$-connected. Therefore $N_{eu}gs\alpha*Cl(A) = 1_N$.

(ii)$\Rightarrow$(iii): Let $A \neq 1_N$ be a $N_{eu}gs\alpha*$-closed set in X. Then A^C is $N_{eu}gs\alpha*$-open set in X and $A^C \neq 0_N$. Hence by hypothesis, $N_{eu}gs\alpha*Cl(A^C) = 1_N$, and so $N_{eu}gs\alpha*Cl(A^C) = \left(N_{eu}gs\alpha*Int(A)\right)^C = 1_N$. This implies that $N_{eu}gs\alpha*Int(A) = 0_N$.

(iii)$\Rightarrow$(iv): Let A and B be $N_{eu}gs\alpha*$-open sets in X such that $A \neq 0_N \neq B$ and $A \subseteq B^C$. Since B^C is $N_{eu}gs\alpha*$-closed set in X and $B \neq 0_N$ implies $B^C \neq 1_N$, we obtain $N_{eu}gs\alpha*Int(B^C) = 0_N$. But, from $A \subseteq B^C$,

$0_N \neq A = N_{eu}gs\alpha*Int(A) \subseteq N_{eu}gs\alpha*Int(B^C) = 0_N$, which is a contradiction.

(iv)$\Rightarrow$(i): Let $0_N \neq A \neq 1_N$ be N_{eu}-Regular-$gs\alpha*$-open set in X. Let $B = \left(N_{eu}gs\alpha*Cl(A)\right)^C$.

Since $N_{eu}gs\alpha*Int\left[N_{eu}gs\alpha*Cl(B)\right] = N_{eu}gs\alpha*Int\left[N_{eu}gs\alpha*Cl\left(N_{eu}gs\alpha*Cl(A)\right)^C\right]$

$= N_{eu}gs\alpha*Int\left[N_{eu}gs\alpha*Int\left(N_{eu}gs\alpha*Cl(A)\right)\right]^C = N_{eu}gs\alpha*Int(A^C) = \left[N_{eu}gs\alpha*Cl(A)\right]^C = B.$

Also we get $B \neq 0_N$, since otherwise, we have $B = 0_N$ and this implies $\left(N_{eu}gs\alpha*Cl(A)\right)^C = 0_N$. That implies

$N_{eu}gs\alpha*Cl(A)=1_N$. That shows that $A=N_{eu}gs\alpha*Int[N_{eu}gs\alpha*Cl(A)]=N_{eu}gs\alpha*Int(1_N)=1_N$. That is $A=1_N$, which is a contradiction. Therefore $B\neq 0_N$ and $A\subseteq B^C$. But this is a contradiction to (iv). Therefore (X,T_N) is N_{eu}-Super-$gs\alpha*$-connected space.

(i)$\Rightarrow$(v): Let A and B be $N_{eu}gs\alpha*$-open sets in (X,T_N) such that $A\neq 0_N\neq B$, $B=[N_{eu}gs\alpha*Cl(A)]^C$, $A=[N_{eu}gs\alpha*Cl(B)]^C$. Now $N_{eu}gs\alpha*Int[N_{eu}gs\alpha*Cl(A)]=N_{eu}gs\alpha*Int(B^C)=[N_{eu}gs\alpha*Cl(B)]^C=A$, $A\neq 0_N$ and $A\neq 1_N$, since if $A=1_N$, then $1_N=[N_{eu}gs\alpha*Cl(B)]^C$. This implies $N_{eu}gs\alpha*Cl(B)=0_N$. But $B\neq 0_N$. Therefore $A\neq 1_N$ implies that A is proper N_{eu}-Regular-$gs\alpha*$-open set in (X,T_N), which is a contradiction to (i). Hence (v) is true.

(v)$\Rightarrow$(i): Let A be N_{eu}-Regular-$gs\alpha*$-open set in (X,T_N) such that $A=N_{eu}gs\alpha*Int[N_{eu}gs\alpha*Cl(A)]$ and $0_N\neq A\neq 1_N$. Now take $B=[N_{eu}gs\alpha*Cl(A)]^C$. In this case we get $B\neq 0_N$ and B is N_{eu}-Regular-$gs\alpha*$-open set in (X,T_N). $B=[N_{eu}gs\alpha*Cl(A)]^C$ and $[N_{eu}gs\alpha*Cl(B)]^C=$ $\left[N_{eu}gs\alpha*Cl\left(N_{eu}gs\alpha*Cl(A)\right)^C\right]^C=$ $N_{eu}gs\alpha*Int\left[\left(N_{eu}gs\alpha*Cl(A)\right)^C\right]^C=N_{eu}gs\alpha*Int\left[\left(N_{eu}gs\alpha*Cl(A)\right)\right]$ $=A$. But this is a contradiction. Therefore (X,T_N) is N_{eu}-Super-$gs\alpha*$-connected space.

(v)$\Rightarrow$(vi): Let A and B be two N_{eu}-Regular-$gs\alpha*$-closed sets in (X,T_N) such that $A\neq 1_N\neq B$, $B=[N_{eu}gs\alpha*Int(A)]^C$, $A=[N_{eu}gs\alpha*Int(B)]^C$. Take $C=A^C$ and $D=B^C$, C and D become N_{eu}-Regular-$gs\alpha*$-open sets in (X,T_N) with $C\neq 0_N\neq D$, $D=[N_{eu}gs\alpha*Int(C)]^C$, $C=[N_{eu}gs\alpha*Int(D)]^C$, which is a contradiction to (v). Hence (vi) is true.

(vi)$\Rightarrow$(v): It can be easily proved by the similar way as in (v)$\Rightarrow$(vi).

Definition 4.13. A N_{eu}-Top-Space (X,T_N) is said to be N_{eu}-Extremely-$gs\alpha*$-disconnected if the $N_{eu}gs\alpha*$-closure of every $N_{eu}gs\alpha*$-open set in (X,T_N) is $N_{eu}gs\alpha*$-open set in X.

Theorem 4.14. Let (X,T_N) be a N_{eu}-Top-Space. Then the following statements are equivalent.

(i) X is N_{eu}-Extremely-$gs\alpha*$-disconnected space.

(ii) For each $N_{eu}gs\alpha*$-closed set A, $N_{eu}gs\alpha*Int(A)$ is $N_{eu}gs\alpha*$-closed set.

(iii) For each $N_{eu}gs\alpha*$-open set A, $N_{eu}gs\alpha*Cl(A)=\left[N_{eu}gs\alpha*Cl\left(N_{eu}gs\alpha*Cl(A)\right)^C\right]^C$.

(iv) For each $N_{eu}gs\alpha*$-open sets A and B with $N_{eu}gs\alpha*Cl(A)=B^C$, $N_{eu}gs\alpha*Cl(A)=[N_{eu}gs\alpha*Cl(B)]^C$.

Proof. (i)$\Rightarrow$(ii): Let A be any $N_{eu}gs\alpha*$-closed set in (X,T_N). Then A^C is $N_{eu}gs\alpha*$-open set. So (i) implies that $N_{eu}gs\alpha*Cl(A^C)=[N_{eu}gs\alpha*Int(A)]^C$ is $N_{eu}gs\alpha*$-open set. Thus $N_{eu}gs\alpha*Int(A)$ is $N_{eu}gs\alpha*$-closed set in (X,T_N).

(ii)$\Rightarrow$(iii): Let A be $N_{eu}gs\alpha*$-open set. Then we have $\left[N_{eu}gs\alpha*Cl\left(N_{eu}gs\alpha*Cl(A)\right)^C\right]^C=$ $\left[N_{eu}gs\alpha*Cl\left(N_{eu}gs\alpha*Int(A^C)\right)\right]^C$. Since A is $N_{eu}gs\alpha*$-open set. Then A^C is $N_{eu}gs\alpha*$-closed set. So, by (ii) $N_{eu}gs\alpha*Int(A^C)$ is $N_{eu}gs\alpha*$-closed set. That is $N_{eu}gs\alpha*Cl[N_{eu}gs\alpha*Int(A^C)]=N_{eu}gs\alpha*Int(A^C)$. Hence we obtain

$$\left[N_{eu}gs\alpha*Cl\left(N_{eu}gs\alpha*Cl(A)\right)^C\right]^C=$$
$$\left[N_{eu}gs\alpha*Cl\left(N_{eu}gs\alpha*Int(A^C)\right)\right]^C=$$
$$\left[N_{eu}gs\alpha*Int(A^C)\right]^C=$$

$N_{eu}gs\alpha^*Cl(A)$ which implies that $N_{eu}gs\alpha^*Cl(A)=\left[N_{eu}gs\alpha^*Cl\left(N_{eu}gs\alpha^*Cl(A)\right)^C\right]^C$.

$(iii)\Rightarrow(iv)$: Let A and B be any two $N_{eu}gs\alpha^*$-open sets in (X,T_N) such that $N_{eu}gs\alpha^*Cl(A)=B^C$. Then $(iii)\Rightarrow N_{eu}gs\alpha^*Cl(A)=$ $\left[N_{eu}gs\alpha^*Cl\left(N_{eu}gs\alpha^*Cl(A)\right)^C\right]^C$ $=\left[N_{eu}gs\alpha^*Cl\left(B^C\right)^C\right]^C=\left[N_{eu}gs\alpha^*Cl(B)\right]^C$.

$(iv)\Rightarrow(i)$: Let A be any $N_{eu}gs\alpha^*$-open set in (X,T_N). Let $B=\left[N_{eu}gs\alpha^*Cl(A)\right]^C$. Then $N_{eu}gs\alpha^*Cl(A)=B^C$. Then (iv) implies $N_{eu}gs\alpha^*Cl(A)=\left[N_{eu}gs\alpha^*Cl(B)\right]^C$. Since $N_{eu}gs\alpha^*Cl(B)$ is $N_{eu}gs\alpha^*$-closed set, this implies that $N_{eu}gs\alpha^*Cl(A)$ is $N_{eu}gs\alpha^*$-open set. This implies that (X,T_N) is N_{eu}-Extremely-$gs\alpha^*$-disconnected space.

Definition 4.15. A N_{eu}-Top-Space. (X,T_N) is N_{eu}-Strongly-$gs\alpha^*$-connected, if there does not exist any nonempty $N_{eu}gs\alpha^*$-closed sets A and B in X such that $A\cap B=0_N$.

Theorem 4.16. Let $f:(X,T_N)\to(Y,\sigma_N)$ be a $N_{eu}gs\alpha^*$-irresolute surjection and X be a N_{eu}-Strongly-$gs\alpha^*$-connected space. Then Y is N_{eu}-Strongly-$gs\alpha^*$-connected.

Proof. Assume that Y is not N_{eu}-Strongly-$gs\alpha^*$-connected, then there exist nonempty $N_{eu}gs\alpha^*$-closed sets U and V in Y such that $U\neq 0_N, V\neq 0_N$, and $U\cap V=0_N$. Since f is $N_{eu}gs\alpha^*$-irresolute mapping, $A=f^{-1}(U)\neq 0_N$, $B=f^{-1}(V)\neq 0_N$, which are $N_{eu}gs\alpha^*$-closed sets in X and $f^{-1}(U)\cap f^{-1}(V)=f^{-1}(0_N)=0_N$, which implies $A\cap B=0_N$. Thus, X is not N_{eu}-Strongly-$gs\alpha^*$-connected, which is a N_{eu}-Strongly-$gs\alpha^*$-connected. Hence Y is contradiction to our hypothesis.

V. NEUTROSOPHIC GENERALIZED SEMI ALPHA STAR REGULAR SPACES

In this section, we define $N_{eu}gs\alpha^*$-Regular spaces and Strongly-$N_{eu}gs\alpha^*$-Regular spaces by using $N_{eu}gs\alpha^*$-open sets and $N_{eu}gs\alpha^*$-closed sets in N_{eu}-Top-Spaces. We study their basic properties and characterizations.

Definition 5.1. A N_{eu}-Top-Space (X,τ_N) is said to be $N_{eu}gs\alpha^*$-Regular if for each $N_{eu}gs\alpha^*$-closed set A and a neutrosophic point $x_{(\alpha,\beta,\gamma)}\notin A$, there exist disjoint $N_{eu}gs\alpha^*$-open sets U and V such that $A\subseteq U$, $x_{(\alpha,\beta,\gamma)}\in V$.

Theorem 5.2. Let (X,τ_N) be a N_{eu}-Top-Space. Then the following statements are equivalent:

(i) X is $N_{eu}gs\alpha^*$-Regular.

(ii) For every $x_{(\alpha,\beta,\gamma)}\in X$ and every $N_{eu}gs\alpha^*$-open set G containing $x_{(\alpha,\beta,\gamma)}$, there exists a $N_{eu}gs\alpha^*$-open set U such that $x_{(\alpha,\beta,\gamma)}\in U\subseteq N_{eu}gs\alpha^*Cl(U)\subseteq G$.

(iii) For every $N_{eu}gs\alpha^*$-closed set F, the intersection of all $N_{eu}gs\alpha^*$-closed $N_{eu}gs\alpha^*$-neighbourhoods of F is exactly F.

(iv) For any neutrosophic set A and a $N_{eu}gs\alpha^*$-open set B such that $A\cap B\neq 0_N$, there exists a $N_{eu}gs\alpha^*$-open set U such that $A\cap U\neq 0_N$ and $N_{eu}gs\alpha^*Cl(U)\subseteq B$.

(v) For every non-empty neutrosophic set A and $N_{eu}gs\alpha^*$-closed set B such that $A\cap B=0_N$, there exist disjoint $N_{eu}gs\alpha^*$-open sets U and V such that $A\cap U\neq 0_N$ and $B\subseteq V$.

Proof. $(i)\Rightarrow(ii)$: Suppose X is $N_{eu}gs\alpha^*$-Regular. Let $x_{(\alpha,\beta,\gamma)}\in X$ and let G be a $N_{eu}gs\alpha^*$-open set containing $x_{(\alpha,\beta,\gamma)}$. Then $x_{(\alpha,\beta,\gamma)}\notin G^C$ and G^C is $N_{eu}gs\alpha^*$-closed. Since X is $N_{eu}gs\alpha^*$-Regular, there exist $N_{eu}gs\alpha^*$-open sets U and V such that $U\cap V=0_N$ and $x_{(\alpha,\beta,\gamma)}\in U$, $G^C\subseteq V$. It follows that $U\subseteq V^C\subseteq G$ and hence $N_{eu}gs\alpha^*Cl(U)\subseteq N_{eu}gs\alpha^*Cl\left(V^C\right)=V^C\subseteq G$. That is $x_{(\alpha,\beta,\gamma)}\in U\subseteq N_{eu}gs\alpha^*Cl(U)\subseteq G$.

$(\text{ii})\Rightarrow(\text{iii})$: Let F be any $N_{eu}gs\alpha*-$closed set and $x_{(\alpha,\beta,\gamma)}\notin F$. Then F^C is $N_{eu}gs\alpha*-$open and $x_{(\alpha,\beta,\gamma)}\in F^C$. By assumption, there exists a $N_{eu}gs\alpha*-$open set U such that $x_{(\alpha,\beta,\gamma)}\in U\subseteq N_{eu}gs\alpha*Cl(U)\subseteq F^C$. Thus $F\subseteq(N_{eu}gs\alpha*Cl(U))^C\subseteq U^C$. Now U^C is $N_{eu}gs\alpha*-$closed and $N_{eu}gs\alpha*-$neighbourhood of F which does not contain $x_{(\alpha,\beta,\gamma)}$. So, we get the intersection of all $N_{eu}gs\alpha*-$closed $N_{eu}gs\alpha*-$neighbourhoods of F to be exactly equal to F.

$(\text{iii})\Rightarrow(\text{iv})$: Suppose $A\cap B\neq 0_N$ and B is $N_{eu}gs\alpha*-$open set. Let $x_{(\alpha,\beta,\gamma)}\in A\cap B$. Since B is $N_{eu}gs\alpha*-$open, B^C is $N_{eu}gs\alpha*-$closed and $x_{(\alpha,\beta,\gamma)}\notin B^C$. By using (iii), there exists a $N_{eu}gs\alpha*-$closed, $N_{eu}gs\alpha*-$neighbourhood V of B^C such that $x_{(\alpha,\beta,\gamma)}\notin V$. Now for the $N_{eu}gs\alpha*-$neighbourhood V of B^C, there exists a $N_{eu}gs\alpha*-$open set G such that $B^C\subseteq G\subseteq V$. Take $U=V^C$. Thus U is a $N_{eu}gs\alpha*-$open set containing $x_{(\alpha,\beta,\gamma)}$. Also $A\cap U\neq 0_N$ and $N_{eu}gs\alpha*Cl(U)\subseteq G^C\subseteq B$.

$(\text{iv})\Rightarrow(\text{v})$: Suppose A is a non-empty set and B is a $N_{eu}gs\alpha*-$closed set such that $A\cap B=0_N$. Then B^C is $N_{eu}gs\alpha*-$open set and $A\cap B^C\neq 0_N$. By our assumption, there exists a $N_{eu}gs\alpha*-$open U such that $A\cap U\neq 0_N$ and $N_{eu}gs\alpha*Cl(U)\subseteq B^C$. Take $V=(N_{eu}gs\alpha*Cl(U))^C$. Since $N_{eu}gs\alpha*Cl(U)$ is $N_{eu}gs\alpha*-$closed, V is $N_{eu}gs\alpha*-$open. Also $B\subseteq V$ and $U\cap V\subseteq N_{eu}gs\alpha*Cl(U)\cap(N_{eu}gs\alpha*Cl(U))^C=0_N$.

$(\text{v})\Rightarrow(\text{i})$: Let S be $N_{eu}gs\alpha*-$closed set and $x_{(\alpha,\beta,\gamma)}\notin S$. Then $S\cap\{x_{(\alpha,\beta,\gamma)}\}=0_N$. By (v), there exist disjoint $N_{eu}gs\alpha*-$open sets U and V such that $U\cap\{x_{(\alpha,\beta,\gamma)}\}\neq 0_N$ and $S\subseteq V$. That is U and V are disjoint $N_{eu}gs\alpha*-$open sets containing $x_{(\alpha,\beta,\gamma)}$ and S respectively. This proves that (X,τ_N) is $N_{eu}gs\alpha*-$Regular.

Corollary 5.3. Let (X,τ_N) be a $N_{eu}-$Top$-$Space. Then the following statements are equivalent:

(i) X is $N_{eu}gs\alpha*-$Regular.

(ii) For every $x_{(\alpha,\beta,\gamma)}\in X$ and every $N_{eu}gs\alpha*-$open set G containing $x_{(\alpha,\beta,\gamma)}$, there exists a $N_{eu}gs\alpha*-$open set U such that $x_{(\alpha,\beta,\gamma)}\in U\subseteq N_{eu}gs\alpha*Cl(U)\subseteq G$.

(iii) For every $N_{eu}gs\alpha*-$closed F, the intersection of all neutrosophic closed, neutrosophic neighbourhoods of F is exactly F.

(iv) For any neutrosophic set A and a neutrosophic open set B such that $A\cap B\neq 0_N$, there exists a $N_{eu}gs\alpha*-$open set U such that $A\cap U\neq 0_N$ and $N_{eu}gs\alpha*Cl(U)\subseteq B$.

(v) For every non-empty neutrosophic set A and a neutrosophic closed set B such that $A\cap B=0_N$, there exist disjoint $N_{eu}gs\alpha*-$open sets U and V such that $A\cap U\neq 0_N$ and $B\subseteq V$.

Proof. Since every neutrosophic open set is $N_{eu}gs\alpha*-$open and follows from Theorem 5.2.

Theorem 5.4. A $N_{eu}-$Top$-$Space (X,τ_N) is $N_{eu}gs\alpha*-$Regular if and only if every $x_{(\alpha,\beta,\gamma)}\in X$ and every $N_{eu}gs\alpha*-$neighbourhood N containing $x_{(\alpha,\beta,\gamma)}$, there exists a $N_{eu}gs\alpha*-$open set V such that $x_{(\alpha,\beta,\gamma)}\in V\subseteq N_{eu}gs\alpha*Cl(V)\subseteq N$.

Proof. Let X be a $N_{eu}gs\alpha*-$Regular space. Let N be any $N_{eu}gs\alpha*-$neighbourhood of $x_{(\alpha,\beta,\gamma)}$. Then there exists a $N_{eu}gs\alpha*-$open set G such that $x_{(\alpha,\beta,\gamma)}\in G\subseteq N$. Since G^C is $N_{eu}gs\alpha*-$closed set and $x_{(\alpha,\beta,\gamma)}\notin G^C$, by definition there exist $N_{eu}gs\alpha*-$open sets U and V such $G^C\subseteq U$ and $x_{(\alpha,\beta,\gamma)}\in V$ and $U\cap V=0_N$ so that $V\subseteq U^C$. It follows that $N_{eu}gs\alpha*Cl(V)\subseteq N_{eu}gs\alpha*Cl(U^C)=U^C$. Also $G^C\subseteq U$ implies $U^C\subseteq G\subseteq N$. Hence $x_{(\alpha,\beta,\gamma)}\in V\subseteq N_{eu}gs\alpha*Cl(V)\subseteq N$. Conversely, suppose for every $x_{(\alpha,\beta,\gamma)}\in X$ and every $N_{eu}gs\alpha*-$neighbourhood N containing $x_{(\alpha,\beta,\gamma)}$, there exists a $N_{eu}gs\alpha*-$open set V such that $x_{(\alpha,\beta,\gamma)}\in V\subseteq N_{eu}gs\alpha*Cl(V)\subseteq N$. Let F be any $N_{eu}gs\alpha*-$closed set and $x_{(\alpha,\beta,\gamma)}\notin F$. Then $x_{(\alpha,\beta,\gamma)}\in F^C$. Since F^C is $N_{eu}gs\alpha*-$open set, F^C is neutrosophic semi $-\alpha-$neighbourhood containing $x_{(\alpha,\beta,\gamma)}$. By hypothesis there exists a $N_{eu}gs\alpha^*-$open set V such that $x_{(\alpha,\beta,\gamma)} \in V$ and $N_{eu}gs\alpha^* Cl(V) \subseteq F^C$. This implies that $F \subseteq (N_{eu}gs\alpha^* Cl(V))^C$. Then $(N_{eu}gs\alpha^* Cl(V))^C$ is a $N_{eu}gs\alpha^*-$open set containing F. Also $V \cap (N_{eu}gs\alpha^* Cl(V))^C = 0_N$. Hence (X,τ_N) is $N_{eu}gs\alpha^*-$Regular.

Theorem 5.5. A $N_{eu}-$Top$-$Space (X,τ_N) is $N_{eu}gs\alpha^*-$Regular if and only if for each $N_{eu}gs\alpha^*-$closed set F of X and each $x_{(\alpha,\beta,\gamma)} \in F^C$, there exist $N_{eu}gs\alpha^*-$open sets U and V of X such that $x_{(\alpha,\beta,\gamma)} \in U$ and $F \subseteq V$ and $N_{eu}gs\alpha^* Cl(U) \cap N_{eu}gs\alpha^* Cl(V) = 0_N$.

Proof. Suppose (X,τ_N) is $N_{eu}gs\alpha^*-$Regular. Let F be a $N_{eu}gs\alpha^*-$closed set in X and $x_{(\alpha,\beta,\gamma)} \notin F$. Then there exist $N_{eu}gs\alpha^*-$open sets $U_{x_{(\alpha,\beta,\gamma)}}$ and V such that $x_{(\alpha,\beta,\gamma)} \in U_{x_{(\alpha,\beta,\gamma)}}$, $F \subseteq V$ and $U_{x_{(\alpha,\beta,\gamma)}} \cap V = 0_N$. This implies that $U_{(x_{(\alpha,\beta,\gamma)})} \cap N_{eu}gs\alpha^* Cl(V) = 0_N$. Also $N_{eu}gs\alpha^* Cl(V)$ is a $N_{eu}gs\alpha^*-$closed set and $x_{(\alpha,\beta,\gamma)} \notin N_{eu}gs\alpha^* Cl(V)$. Since (X,τ_N) is $N_{eu}gs\alpha^*-$Regular, there exist $N_{eu}gs\alpha^*-$open sets G and H of X such that $x_{(\alpha,\beta,\gamma)} \in G$, $N_{eu}gs\alpha^* Cl(V) \subseteq H$ and $G \cap V = 0_N$. This implies $N_{eu}gs\alpha^* Cl(G) \cap H \subseteq N_{eu}gs\alpha^* Cl(H^C) \cap H = H^C \cap H = 0_N$. Take $U = G$. Now U and V are $N_{eu}gs\alpha^*-$open sets in X such that $x_{(\alpha,\beta,\gamma)} \in U$ and $F \subseteq V$. Also $N_{eu}gs\alpha^* Cl(U) \cap N_{eu}gs\alpha^* Cl(V) \subseteq N_{eu}gs\alpha^* Cl(G) \cap H = 0_N$.

Conversely, suppose for each $N_{eu}gs\alpha^*-$closed set F of X and each $x_{(\alpha,\beta,\gamma)} \in F^C$, there exist $N_{eu}gs\alpha^*-$open sets U and V of X such that $x_{(\alpha,\beta,\gamma)} \in U$ and $F \subseteq V$ and $N_{eu}gs\alpha^* Cl(U) \cap N_{eu}gs\alpha^* Cl(V) = 0_N$. Now $U \cap V \subseteq N_{eu}gs\alpha^* Cl(U) \cap N_{eu}gs\alpha^* Cl(V) = 0_N$. Therefore $U \cap V = 0_N$. This proves that (X,τ_N) is $N_{eu}gs\alpha^*-$Regular.

Theorem 5.6. Let $f:(X,\tau_N) \to (Y,\sigma_N)$ be a bijective function. If f is $N_{eu}gs\alpha^*-$irresolute, $N_{eu}gs\alpha^*-$open and X is $N_{eu}gs\alpha^*-$Regular, then Y is $N_{eu}gs\alpha^*-$Regular.

Proof. Suppose (X,τ_N) is $N_{eu}gs\alpha^*-$Regular. Let S be any $N_{eu}gs\alpha^*-$closed set in Y such that $y_{(r,t,s)} \notin S$. Since f is $N_{eu}gs\alpha^*-$irresolute, $f^{-1}(S)$ is $N_{eu}gs\alpha^*-$closed set in X. Since f is onto, there exists $x_{(\alpha,\beta,\gamma)} \in X$ such that $y_{(r,t,s)} = f(x_{(\alpha,\beta,\gamma)})$. Now $f(x_{(\alpha,\beta,\gamma)}) = y_{(r,t,s)} \notin S$ implies that $x_{(\alpha,\beta,\gamma)} \notin f^{-1}(S)$. Since X is $N_{eu}gs\alpha^*-$Regular, there exist $N_{eu}gs\alpha^*-$open sets U and V in X such that $x_{(\alpha,\beta,\gamma)} \in U$, $f^{-1}(S) \subseteq V$ and $U \cap V = 0_N$. Now $x_{(\alpha,\beta,\gamma)} \in U$ implies that $f(x_{(\alpha,\beta,\gamma)}) \in f(U)$ and $f^{-1}(S) \subseteq V$ implies that $S \subseteq f(V)$. Also $U \cap V = 0_N$ implies that $f(U \cap V) = 0_N$ which implies that $f(U) \cap f(V) = 0_N$. Since f is a $N_{eu}gs\alpha^*-$open mapping, $f(U)$ and $f(V)$ are disjoint $N_{eu}gs\alpha^*-$open sets in Y containing $y_{(r,t,s)}$ and S respectively. Thus Y is $N_{eu}gs\alpha^*-$Regular.

Theorem 5.7. Let (X,τ_N) be a $N_{eu}gs\alpha^*-$Regular space. Then

(i) Every $N_{eu}gs\alpha^*-$open set in X is a union of $N_{eu}gs\alpha^*-$closed sets.

(ii) Every $N_{eu}gs\alpha^*-$closed set in X is an intersection of $N_{eu}gs\alpha^*-$open sets.

Proof. (i) Suppose X is $N_{eu}gs\alpha^*-$Regular. Let G be a $N_{eu}gs\alpha^*-$open set and $x_{(\alpha,\beta,\gamma)} \in G$. Then $F = G^C$ is $N_{eu}gs\alpha^*-$closed set and $x_{(\alpha,\beta,\gamma)} \notin F$. Since X is $N_{eu}gs\alpha^*-$Regular, there exist disjoint $N_{eu}gs\alpha^*-$open sets $U_{x_{(\alpha,\beta,\gamma)}}$ and V in X such that $x_{(\alpha,\beta,\gamma)} \in U_{x_{(\alpha,\beta,\gamma)}}$ and $F \subseteq V$. Since $U_{x_{(\alpha,\beta,\gamma)}} \cap F \subseteq U_{x_{(\alpha,\beta,\gamma)}} \cap V = 0_N$, we have $U_{x_{(\alpha,\beta,\gamma)}} \subseteq F^C = G$. Take $V_{(x_{(\alpha,\beta,\gamma)})} = N_{eu}gs\alpha^* Cl(U_{x_{(\alpha,\beta,\gamma)}})$. Then $V_{x_{(\alpha,\beta,\gamma)}}$ is $N_{eu}gs\alpha^*-$closed set and $V_{x_{(\alpha,\beta,\gamma)}} \cap V = 0_N$. Now $F \subseteq V$ implies that $V_{x_{(\alpha,\beta,\gamma)}} \cap F \subseteq V_{x_{(\alpha,\beta,\gamma)}} \cap V = 0_N$. It follows that $x_{(\alpha,\beta,\gamma)} \in V_{x_{(\alpha,\beta,\gamma)}} \subseteq F^C = G$. This proves that

$G=\bigcup\left\{V_{x_{(\alpha,\beta,\gamma)}}:x_{(\alpha,\beta,\gamma)}\in G\right\}$. Thus G is a union of $N_{eu}gs\alpha*$-closed sets. (ii) Follows from (i) and set theoretic properties.

Theorem 5.8. Let $f:(X,\tau_N)\to(Y,\sigma_N)$ be a $N_{eu}gs\alpha*$-continuous and neutrosophic closed injection from a N_{eu}-Top-Space (X,τ_N) into a neutrosophic regular space (Y,σ_N). If every $N_{eu}gs\alpha*$-closed set in X is neutrosophic closed, then X is $N_{eu}gs\alpha*$-Regular.

Proof. Let $x_{(\alpha,\beta,\gamma)}\in X$ and A be a $N_{eu}gs\alpha*$-closed set in X such that $x_{(\alpha,\beta,\gamma)}\notin A$. Then by assumption, A is neutrosophic closed in X. Since f is neutrosophic closed, $f(A)$ is a neutrosophic closed set in Y such that $f\left(x_{(\alpha,\beta,\gamma)}\right)\notin f(A)$. Since Y is neutrosophic regular, there exist disjoint neutrosophic open sets G and H in Y such that $f\left(x_{(\alpha,\beta,\gamma)}\right)\in G$ and $f(A)\subseteq H$. Since f is $N_{eu}gs\alpha*$-continuous, $f^{-1}(G)$ and $f^{-1}(H)$ are disjoint $N_{eu}gs\alpha*$-open sets in X containing $x_{(\alpha,\beta,\gamma)}$ and A respectively. Hence X is $N_{eu}gs\alpha*$-Regular.

Theorem 5.9. Let $f:(X,\tau_N)\to(Y,\sigma_N)$ be a neutrosophic continuous, $N_{eu}gs\alpha*$-open bijection of a neutrosophic regular space X into a neutrosophic space Y and if every $N_{eu}gs\alpha*$-closed set in Y is neutrosophic closed, then Y is $N_{eu}gs\alpha*$-Regular.

Proof. Let $y_{(r,t,s)}\in Y$ and B be a $N_{eu}gs\alpha*$-closed set in Y such that $y_{(r,t,s)}\notin B$. Since f is a bijection. So there exists a unique point $x_{(\alpha,\beta,\gamma)}\in X$ such that $f\left(x_{(\alpha,\beta,\gamma)}\right)=y_{(r,t,s)}$. Then by assumption, B is neutrosophic closed in Y. Since f is a neutrosophic continuous bijection, $f^{-1}(B)$ is a neutrosophic closed set in X such that $x_{(\alpha,\beta,\gamma)}\notin f^{-1}(B)$. Since X is neutrosophic regular, there exist disjoint neutrosophic open sets G and H in X such that $x_{(\alpha,\beta,\gamma)}\in G$ and $f^{-1}(B)\subseteq H$. Since f is $N_{eu}gs\alpha*$-open, $f(G)$ and $f(H)$ are disjoint $N_{eu}gs\alpha*$-open sets in Y such that $f\left(x_{(\alpha,\beta,\gamma)}\right)=y_{(r,t,s)}\in f(G)$ and $B\subseteq f(H)$. Hence Y is $N_{eu}gs\alpha*$-Regular.

Definition 5.10. A N_{eu}-Top-Space (X,τ_N) is said to be strongly $N_{eu}gs\alpha*$-Regular if for each $N_{eu}gs\alpha*$-closed set A and a point $x_{(\alpha,\beta,\gamma)}\notin A$, there exist disjoint neutrosophic open sets U and V such that $A\subseteq U$ and $x_{(\alpha,\beta,\gamma)}\in V$.

Proposition 5.11. (i) Every strongly $N_{eu}gs\alpha*$-Regular space is $N_{eu}gs\alpha*$-Regular.

(ii) Every strongly $N_{eu}gs\alpha*$-Regular space is strongly neutrosophic regular.

Proof. (i) Suppose (X,τ_N) is strongly $N_{eu}gs\alpha*$-Regular. Let F be a $N_{eu}gs\alpha*$-closed set and $x_{(\alpha,\beta,\gamma)}\notin F$. Since X is strongly $N_{eu}gs\alpha*$-Regular, there exist disjoint neutrosophic open sets U and V such that $x_{(\alpha,\beta,\gamma)}\in U$ and $F\subseteq V$. Since every neutrosophic open set is $N_{eu}gs\alpha*$-open, so U and V are $N_{eu}gs\alpha*$-open sets. This implies that X is $N_{eu}gs\alpha*$-Regular.

(ii) This can be proved similarly as (i).

Definition 5.12. A N_{eu}-Top-Space (X,τ_N) is said to be strongly* $N_{eu}gs\alpha*$-Regular. if for each neutrosophic closed set A and a point $x_{(\alpha,\beta,\gamma)}\notin A$, there exist disjoint $N_{eu}gs\alpha*$-open sets U and V such that $A\subseteq U$, $x_{(\alpha,\beta,\gamma)}\in V$.

Proposition 5.13. Every $N_{eu}gs\alpha*$-Regular N_{eu}-Top-Space (X,τ_N) is strongly* $N_{eu}gs\alpha*$-Regular.

Proof. Suppose (X,τ_N) is $N_{eu}gs\alpha*$-Regular. Let F be a neutrosophic closed set and $x_{(\alpha,\beta,\gamma)}\notin F$. Then F is $N_{eu}gs\alpha*$-closed. Since X is $N_{eu}gs\alpha*$-Regular, there exist disjoint $N_{eu}gs\alpha*$-open sets U and V such that $x_{(\alpha,\beta,\gamma)}\in U$ and $F\subseteq V$. This implies that X is strongly* $N_{eu}gs\alpha*$-Regular.

Theorem 5.14. Let (X,τ_N) be a N_{eu}-Top-Space. Then the following statements are equivalent:

(i) X is strongly $N_{eu}gs\alpha*$-Regular.

(ii) For every $x_{(\alpha,\beta,\gamma)}\in X$ and every $N_{eu}gs\alpha*$-open set G containing $x_{(\alpha,\beta,\gamma)}$, there exists a neutrosophic open set U such that $x_{(\alpha,\beta,\gamma)}\in U\subseteq N_{eu}Cl(U)\subseteq G$.

(iii) For every $N_{eu}gs\alpha*$-closed set F, the intersection of all neutrosophic closed, neutrosophic neighborhoods of F is exactly F.

(iv) For any neutrosophic set A and a $\mathrm{N}_{eu}gs\alpha*$-open set B such that $A \cap B \neq 0_N$, there exists a neutrosophic open set U such that $A \cap U \neq 0_N$ and $\mathrm{N}_{eu}Cl(U) \subseteq B$.

(v) For every non-empty neutrosophic set A and $\mathrm{N}_{eu}gs\alpha*$-closed set B such that $A \cap B = 0_N$, there exist disjoint neutrosophic open sets U and V such that $A \cap U \neq 0_N$ and $B \subseteq V$.

Proof. $(i) \Rightarrow (ii)$: Suppose X is strongly $\mathrm{N}_{eu}gs\alpha*$-Regular. Let $x_{(\alpha,\beta,\gamma)} \in X$ and let G be a $\mathrm{N}_{eu}gs\alpha*$-open set containing $x_{(\alpha,\beta,\gamma)}$. Then $x_{(\alpha,\beta,\gamma)} \notin G^C$ and G^C is $\mathrm{N}_{eu}gs\alpha*$-closed. Since X is $\mathrm{N}_{eu}gs\alpha*$-Regular, there exist neutrosophic open sets U and V such that $U \cap V = 0_N$ and $x_{(\alpha,\beta,\gamma)} \in U$, $G^C \subseteq V$. It follows that $U \subseteq V^C \subseteq G$ and hence $\mathrm{N}_{eu}Cl(U) \subseteq \mathrm{N}_{eu}Cl(V^C) = V^C \subseteq G$. That is $x_{(\alpha,\beta,\gamma)} \in U \subseteq \mathrm{N}_{eu}Cl(U) \subseteq G$.

$(ii) \Rightarrow (iii)$: Let F be a $\mathrm{N}_{eu}gs\alpha*$-closed set and $x_{(\alpha,\beta,\gamma)} \notin F$. Then F^C is $\mathrm{N}_{eu}gs\alpha*$-open set and $x_{(\alpha,\beta,\gamma)} \in F^C$. By assumption, there exists a neutrosophic open set U such that $x_{(\alpha,\beta,\gamma)} \in U \subseteq \mathrm{N}_{eu}Cl(U) \subseteq F^C$. Thus $F \subseteq (\mathrm{N}_{eu}Cl(U))^C \subseteq U^C$. Now U^C is neutrosophic closed, neutrosophic neighborhood of F which does not contain $x_{(\alpha,\beta,\gamma)}$. So, the intersection of all neutrosophic closed, neutrosophic neighborhoods of F is exactly F.

$(iii) \Rightarrow (iv)$: Suppose $A \cap B \neq 0_N$ and B is $\mathrm{N}_{eu}gs\alpha*$-open set. Let $x_{(\alpha,\beta,\gamma)} \in A \cap B$. Since B is $\mathrm{N}_{eu}gs\alpha*$-open, B^C is $\mathrm{N}_{eu}gs\alpha*$-closed and $x_{(\alpha,\beta,\gamma)} \notin B^C$. By using (iii), there exists a neutrosophic closed, neutrosophic neighborhood V of B^C such that $x_{(\alpha,\beta,\gamma)} \notin V$. Now for the neutrosophic neighbourhood V of B^C, there exists a neutrosophic open set G such that $B^C \subseteq G \subseteq V$. Take $U = V^C$. Thus U is a neutrosophic open set containing $x_{(\alpha,\beta,\gamma)}$. Also $A \cap U \neq 0_N$ and $\mathrm{N}_{eu}Cl(U) \subseteq G^C \subseteq B$.

$(iv) \Rightarrow (v)$: Suppose A is a non-empty set and B is $\mathrm{N}_{eu}gs\alpha*$-closed set such that $A \cap B = 0_N$. Then B^C is $\mathrm{N}_{eu}gs\alpha*$-open set and $A \cap B^C \neq 0_N$. By our assumption, there exists a neutrosophic open set U such that $A \cap U \neq 0_N$ and $\mathrm{N}_{eu}Cl(U) \subseteq B^C$. Take $V = (\mathrm{N}_{eu}Cl(U))^C$. Since $\mathrm{N}_{eu}Cl(U)$ is neutrosophic closed, V is neutrosophic open. Also $B \subseteq V$ and $U \cap V \subseteq \mathrm{N}_{eu}Cl(U) \cap (\mathrm{N}_{eu}Cl(U))^C = 0_N$.

$(v) \Rightarrow (i)$: Let S be $\mathrm{N}_{eu}gs\alpha*$-closed set and $x_{(\alpha,\beta,\gamma)} \notin S$. Then $S \cap \{x_{(\alpha,\beta,\gamma)}\} = 0_N$. By (v), there exist disjoint neutrosophic open sets U and V such that $U \cap \{x_{(\alpha,\beta,\gamma)}\} \neq 0_N$ and $S \subseteq V$. That is U and V are disjoint neutrosophic open sets containing $x_{(\alpha,\beta,\gamma)}$ and S respectively. This proves that (X, τ_N) is strongly $\mathrm{N}_{eu}gs\alpha*$-Regular.

Theorem 5.15. A N_{eu}-Top-Space. (X, τ_N) is $\mathrm{N}_{eu}gs\alpha*$-Regular if and only if for each $\mathrm{N}_{eu}gs\alpha*$-closed set F of X and each $x_{(\alpha,\beta,\gamma)} \in F^C$, there exist neutrosophic open sets U and V of X such that $x_{(\alpha,\beta,\gamma)} \in U$ and $F \subseteq V$ and $\mathrm{N}_{eu}Cl(U) \cap \mathrm{N}_{eu}Cl(V) = 0_N$.

Proof. Suppose (X, τ_N) is strongly $\mathrm{N}_{eu}gs\alpha*$-Regular. Let F be a $\mathrm{N}_{eu}gs\alpha*$-closed set in X and $x_{(\alpha,\beta,\gamma)} \notin F$. Then there exist neutrosophic open sets $U_{x_{(\alpha,\beta,\gamma)}}$ and V such that $x_{(\alpha,\beta,\gamma)} \in U_{x_{(\alpha,\beta,\gamma)}}$, $F \subseteq V$ and $U_{x_{(\alpha,\beta,\gamma)}} \cap V = 0_N$. This implies that $U_{(x_{(\alpha,\beta,\gamma)})} \cap \mathrm{N}_{eu}Cl(V) = 0_N$. Also $\mathrm{N}_{eu}Cl(V)$ is a neutrosophic closed set and $x_{(\alpha,\beta,\gamma)} \notin \mathrm{N}_{eu}Cl(V)$. Since (X, τ_N) is strongly $\mathrm{N}_{eu}gs\alpha*$-Regular, there exist neutrosophic open sets G and H of X such that $x_{(\alpha,\beta,\gamma)} \in G$, $\mathrm{N}_{eu}Cl(V) \subseteq H$ and $G \cap V = 0_N$. This implies

$\mathrm{N}_{eu}Cl(G) \cap H \subseteq \mathrm{N}_{eu}Cl(H^C) \cap H = H^C \cap H = 0_N$.

Take $U = G$. Now U and V are neutrosophic open sets in X such that $x_{(\alpha,\beta,\gamma)} \in U$ and $F \subseteq V$. Also $\mathrm{N}_{eu}Cl(U) \cap \mathrm{N}_{eu}Cl(V) \subseteq \mathrm{N}_{eu}Cl(G) \cap H = 0_N$. Thus $\mathrm{N}_{eu}Cl(U) \cap \mathrm{N}_{eu}Cl(V) = 0_N$.

Conversely, suppose for each $\mathrm{N}_{eu}gs\alpha*$-closed set F of X and each $x_{(\alpha,\beta,\gamma)} \in F^C$, there exist neutrosophic open sets U and V of X such that $x_{(\alpha,\beta,\gamma)} \in U$ and $F \subseteq V$ and $\mathrm{N}_{eu}Cl(U) \cap \mathrm{N}_{eu}Cl(V) = 0_N$. Now

$U \cap V \subseteq N_{eu}Cl(U) \cap N_{eu}Cl(V) = 0_N$. Therefore $U \cap V = 0_N$. This proves that (X, τ_N) is strongly $N_{eu}gs\alpha^*$-Regular.

Theorem 5.16. A N_{eu}-Top-Space. (X, τ_N) is strongly $N_{eu}gs\alpha^*$-Regular if and only if every pair consisting of a neutrosophic compact set and a disjoint $N_{eu}gs\alpha^*$-closed set can be separated by neutrosophic open sets.

Proof. Let (X, τ_N) be strongly $N_{eu}gs\alpha^*$-Regular and let A be a neutrosophic compact set, and B be a $N_{eu}gs\alpha^*$-closed set such that $A \cap B = 0_N$. Since X is strongly $N_{eu}gs\alpha^*$-Regular, for each $x_{(\alpha,\beta,\gamma)} \in A$, there exist disjoint neutrosophic open sets $U_{x_{(\alpha,\beta,\gamma)}}$ and $V_{x_{(\alpha,\beta,\gamma)}}$ such that $x_{(\alpha,\beta,\gamma)} \in U_{x_{(\alpha,\beta,\gamma)}}$, $B \subseteq V_{x_{(\alpha,\beta,\gamma)}}$. Obviously, $\{U_{x_{(\alpha,\beta,\gamma)}} : x_{(\alpha,\beta,\gamma)} \in A\}$ is a neutrosophic open covering of A. Since A is neutrosophic compact, there exists a finite set $F \subseteq A$ such that $A \subseteq \bigcup\{U_{x_{(\alpha,\beta,\gamma)}} : x_{(\alpha,\beta,\gamma)} \in F\}$ and $B \subseteq \bigcap\{V_{x_{(\alpha,\beta,\gamma)}} : x_{(\alpha,\beta,\gamma)} \in F\}$. Put $U = \bigcup\{U_{x_{(\alpha,\beta,\gamma)}} : x_{(\alpha,\beta,\gamma)} \in F\}$ and $V = \bigcap\{V_{x_{(\alpha,\beta,\gamma)}} : x_{(\alpha,\beta,\gamma)} \in F\}$. Then U and V are neutrosophic open sets in X. Also $U \cap V = 0_N$. Otherwise, if $x_{(\alpha,\beta,\gamma)} \in U \cap V$, then $x_{(\alpha,\beta,\gamma)} \in U_{(r,s,t)}$ for some $x_{(r,s,t)} \in F$ and $x_{(\alpha,\beta,\gamma)} \in V \subseteq V_{(r,s,t)}$. This implies that $x_{(\alpha,\beta,\gamma)} \in U_{x_{(r,s,t)}} \cap V_{x_{(r,s,t)}}$, which is a contradiction to $U_{x_{(r,s,t)}} \cap V_{x_{(r,s,t)}} = \phi$. Thus U and V are disjoint neutrosophic open sets containing A and B respectively.

Conversely, suppose every pair consisting of a neutrosophic compact set and a disjoint $N_{eu}gs\alpha^*$-closed set can be separated by neutrosophic open sets. Let F be a $N_{eu}gs\alpha^*$-closed set and $x_{(\alpha,\beta,\gamma)} \notin F$. Then $\{x_{(\alpha,\beta,\gamma)}\}$ is neutrosophic compact set of X and $\{x_{(\alpha,\beta,\gamma)}\} \cap F = 0_N$. By our assumption, there exist disjoint neutrosophic open sets U and V such that $x_{(\alpha,\beta,\gamma)} \in U$ and $F \subseteq V$. This proves that X is strongly $N_{eu}gs\alpha^*$-Regular.

Corollary 5.17. If X is a strongly $N_{eu}gs\alpha^*$-Regular space, A is a neutrosophic compact subset of X and B is a $N_{eu}gs\alpha^*$-open set containing A, then there exists a neutrosophic regular open set V such that $A \subseteq V \subseteq N_{eu}Cl(V) \subseteq B$.

Proof. Let X be strongly $N_{eu}gs\alpha^*$-Regular and let A be a neutrosophic compact set, and B be $N_{eu}gs\alpha^*$-open set with $A \subseteq B$. Then B^C is $N_{eu}gs\alpha^*$-closed set such that $B^C \cap A = 0_N$. Since X is a strongly $N_{eu}gs\alpha^*$-Regular space, then there exist disjoint neutrosophic open sets G and H such that $A \subseteq G$ and $B^C \subseteq H$. Take $V = N_{eu}Int[N_{eu}Cl(G)]$. Then $N_{eu}Cl(V) = N_{eu}Cl[N_{eu}Int(N_{eu}Cl(G))] \subseteq N_{eu}Cl[N_{eu}Cl(G)] = N_{eu}Cl(G)$. Since G is a neutrosophic open set and $G \subseteq N_{eu}Cl(G)$, we have $G = N_{eu}Int(G) \subseteq N_{eu}Int[N_{eu}Cl(G)] = V$. This implies that $N_{eu}Cl(G) \subseteq N_{eu}Cl(V)$. It follows that $N_{eu}Cl(V) = N_{eu}Cl(G)$ and $N_{eu}Int[N_{eu}Cl(V)] = N_{eu}Int[N_{eu}Cl(G)] = V$. Thus V is neutrosophic regular open. Now $A \subseteq G = N_{eu}Int(G) \subseteq N_{eu}Int[(N_{eu}Cl(G))] = V$. This implies that $A \subseteq V$ and $N_{eu}Cl(V) = N_{eu}Cl(G) \subseteq H^C \subseteq B$ implies that $A \subseteq V \subseteq N_{eu}Cl(V) \subseteq B$.

Theorem 5.18. Let $f : (X, \tau_N) \to (Y, \sigma_N)$ be a bijective function. If f is $N_{eu}gs\alpha^*$-irresolute, neutrosophic open and X is strongly $N_{eu}gs\alpha^*$-Regular, then Y is strongly $N_{eu}gs\alpha^*$-Regular.

Proof. Suppose (X, τ_N) is strongly $N_{eu}gs\alpha^*$-Regular. Let S be a $N_{eu}gs\alpha^*$-closed set in Y such that $y_{(r,t,s)} \notin S$. Since f is $N_{eu}gs\alpha^*$-irresolute, $f^{-1}(S)$ is $N_{eu}gs\alpha^*$-closed set in X. Since f is onto, there exists $x_{(\alpha,\beta,\gamma)} \in X$ such that $y_{(r,t,s)} = f(x_{(\alpha,\beta,\gamma)})$. Now $f(x_{(\alpha,\beta,\gamma)}) = y_{(r,t,s)} \notin S$ implies that $x_{(\alpha,\beta,\gamma)} \notin f^{-1}(S)$. Since X is strongly $N_{eu}gs\alpha^*$-Regular. there exist neutrosophic open sets U and V in X such that $x_{(\alpha,\beta,\gamma)} \in U$, $f^{-1}(S) \subseteq V$ and $U \cap V = 0_N$. Now $x_{(\alpha,\beta,\gamma)} \in U$ implies that $f(x_{(\alpha,\beta,\gamma)}) \in f(U)$ and $f^{-1}(S) \subseteq V$ implies that $S \subseteq f(V)$. Also $U \cap V = 0_N$ implies that $f(U \cap V) = 0_N$ which implies that $f(U) \cap f(V) = 0_N$. Since f is a neutrosophic open mapping, $f(U)$ and $f(V)$ are disjoint neutrosophic open sets in Y containing $y_{(r,t,s)}$ and S respectively. Thus Y is strongly $N_{eu}gs\alpha*-$Regular.

VI. NEUTROSOPHIC GENERALIZED SEMI ALPHA STAR NORMAL SPACES

In this section, we introduce $N_{eu}gs\alpha*-$Normal and strongly $N_{eu}gs\alpha*-$Normal spaces and study their properties and characteristics.

Definition 6.1. A $N_{eu}-$Top$-$Space (X,T_N) is said to be $N_{eu}gs\alpha*-$Normal if for any two disjoint $N_{eu}gs\alpha*-$closed sets A and B, there exist disjoint $N_{eu}gs\alpha*-$open sets U and V such that $A\subseteq U$ and $B\subseteq V$.

Theorem 6.2. Let (X,T_N) be a $N_{eu}-$Top$-$Space. Then the following statements are equivalent:

(a) X is $N_{eu}gs\alpha*-$Normal.

(b) For every $N_{eu}gs\alpha*-$closed set A in X and every $N_{eu}gs\alpha*-$open set U containing A, there exists a $N_{eu}gs\alpha*-$open set V containing A such that $N_{eu}gs\alpha*Cl(V)\subseteq U$.

(c) For each pair of disjoint $N_{eu}gs\alpha*-$closed sets A and B in X, there exists a $N_{eu}gs\alpha*-$open set U containing A such that $N_{eu}gs\alpha*Cl(U)\cap B=0_N$.

(d) For each pair of disjoint $N_{eu}gs\alpha*-$closed sets A and B in X, there exist $N_{eu}gs\alpha*-$open sets U and V containing A and B respectively such that $N_{eu}gs\alpha*Cl(U)\cap N_{eu}gs\alpha*Cl(V)=0_N$.

Proof. $(a)\Rightarrow(b)$: Let U be a $N_{eu}gs\alpha*-$open set containing the $N_{eu}gs\alpha*-$closed set A. Then $B=U^C$ is a $N_{eu}gs\alpha*-$closed set disjoint from A. Since X is $N_{eu}gs\alpha*-$Normal, there exist disjoint $N_{eu}gs\alpha*-$open sets V and W containing A and B respectively. Then $N_{eu}gs\alpha*Cl(V)$ is disjoint from B. Since if $y_{(r,t,s)}\in B$, the set W is a $N_{eu}gs\alpha*-$open set containing $y_{(r,t,s)}\in B$ disjoint from V. Hence $N_{eu}gs\alpha*Cl(V)\subseteq U$.

$(b)\Rightarrow(c)$: Let A and B be disjoint $N_{eu}gs\alpha*-$closed sets in X. Then B^C is a $N_{eu}gs\alpha*-$open set containing A. By (b), there exists a $N_{eu}gs\alpha*-$open set U containing A such that $N_{eu}gs\alpha*Cl(U)\subseteq B^C$. Hence $N_{eu}gs\alpha*Cl(U)\cap B=0_N$. This proves (c).

$(c)\Rightarrow(d)$: Let A and B be disjoint $N_{eu}gs\alpha*-$closed sets in X. Then by (c), there exists a $N_{eu}gs\alpha*-$open set U containing A such that $N_{eu}gs\alpha*Cl(U)\cap B=0_N$. Since $N_{eu}gs\alpha*Cl(U)$ is $N_{eu}gs\alpha*-$closed, B and $N_{eu}gs\alpha*Cl(U)$ are disjoint $N_{eu}gs\alpha*-$closed sets in X. Again by (c), there exists a $N_{eu}gs\alpha*-$open set V containing B such that $N_{eu}gs\alpha*Cl(U)\cap N_{eu}gs\alpha*Cl(V)=0_N$. This proves (d).

$(d)\Rightarrow(a)$: Let A and B be disjoint $N_{eu}gs\alpha*-$closed sets in X. By (d), there exist $N_{eu}gs\alpha*-$open sets U and V containing A and B respectively such that $N_{eu}gs\alpha*Cl(U)\cap N_{eu}gs\alpha*Cl(V)=0_N$. Since $U\cap V\subseteq N_{eu}gs\alpha*Cl(U)\cap N_{eu}gs\alpha*Cl(V)$, U and V are disjoint $N_{eu}gs\alpha*-$open sets containing A and B respectively. Hence the result of (a) follows.

Theorem 6.3. A $N_{eu}-$Top$-$Space (X,T_N) is $N_{eu}gs\alpha*-$Normal if and only if for every $N_{eu}gs\alpha*-$closed set F and $N_{eu}gs\alpha*-$open set W containing F, there exists a $N_{eu}gs\alpha*-$open set U such that $F\subseteq U\subseteq N_{eu}gs\alpha*Cl(U)\subseteq W$.

Proof. Let (X,T_N) be $N_{eu}gs\alpha*-$Normal. Let F be a $N_{eu}gs\alpha*-$closed set and let W be a $N_{eu}gs\alpha*-$open set containing F. Then F and W^C are disjoint $N_{eu}gs\alpha*-$closed sets. Since X is $N_{eu}gs\alpha*-$Normal, there exist disjoint $N_{eu}gs\alpha*-$open sets U and V such that $F\subseteq U$ and $W^C\subseteq V$. Thus $F\subseteq U\subseteq V^C\subseteq W$. Since V^C is $N_{eu}gs\alpha*-$closed, so $N_{eu}gs\alpha*Cl(U)\subseteq N_{eu}gs\alpha*Cl(V^C)=V^C\subseteq W$. Thus $F\subseteq U\subseteq N_{eu}gs\alpha*Cl(U)\subseteq W$.

Conversely, suppose the condition holds. Let G and H be two disjoint $N_{eu}gs\alpha*-$closed sets in X. Then H^C is a $N_{eu}gs\alpha*-$open set containing G. By assumption, there exists a $N_{eu}gs\alpha*-$open set U such that $G\subseteq U\subseteq N_{eu}gs\alpha*Cl(U)\subseteq H^C$. Since U is $N_{eu}gs\alpha*-$open and $N_{eu}gs\alpha*Cl(U)$ is $N_{eu}gs\alpha*-$closed. Then $\left(N_{eu}gs\alpha*Cl(U)\right)^C$ is $N_{eu}gs\alpha*-$open. Now $N_{eu}gs\alpha*Cl(U)\subseteq H^C$ implies that $H\subseteq\left(N_{eu}gs\alpha*Cl(U)\right)^C$. Also

$U\cap\left(N_{eu}g s\alpha^{*}Cl(U)\right)^{C}\subseteq N_{eu}g s\alpha^{*}Cl(U)\cap\left(N_{eu}g s\alpha^{*}Cl(U)\right)^{C}=0_{N}$.

That is U and $\left(N_{eu}g s\alpha^{*}Cl(U)\right)^{C}$ are disjoint $N_{eu}gs\alpha^{*}$-open sets containing G and H respectively. This shows that (X,T_N) is $N_{eu}gs\alpha^{*}$-Normal.

Theorem 6.4. Let (X,T_N) be a N_{eu}-Top-Space. Then the following statements are equivalent:

(a) X is $N_{eu}gs\alpha^{*}$-Normal.

(b) For any two $N_{eu}gs\alpha^{*}$-open sets U and V whose union is 1_N, there exist $N_{eu}gs\alpha^{*}$-closed subsets A of U and B of V such that $A\cup B=1_N$.

Proof. $(a)\Rightarrow(b)$: Let U and V be two $N_{eu}gs\alpha^{*}$-open sets in a $N_{eu}gs\alpha^{*}$-Normal space X such that $U\cup V=1_N$. Then U^C and V^C are disjoint $N_{eu}gs\alpha^{*}$-closed sets. Since X is $N_{eu}gs\alpha^{*}$-Normal, then there exist disjoint $N_{eu}gs\alpha^{*}$-open sets G and H such that $U^C\subseteq G$ and $V^C\subseteq H$. Let $A=G^C$ and $B=H^C$. Then A and B are $N_{eu}gs\alpha^{*}$-closed subsets of U and V respectively such that $A\cup B=1_N$. This proves (b).

$(b)\Rightarrow(a)$: Let A and B be disjoint $N_{eu}gs\alpha^{*}$-closed sets in X. Then A^C and B^C are $N_{eu}gs\alpha^{*}$-open sets whose union is 1_N. By (b), there exist $N_{eu}gs\alpha^{*}$-closed sets E and F such that $E\subseteq A^C$, $F\subseteq B^C$ and $E\cup F=1_N$. Then E^C and F^C are disjoint $N_{eu}gs\alpha^{*}$-open sets containing A and B respectively. Therefore X is $N_{eu}gs\alpha^{*}$-Normal.

Definition 6.5. A N_{eu}-Top-Space (X,T_N) is said to be strongly $N_{eu}gs\alpha^{*}$-Normal if for every pair of disjoint neutrosophic closed sets A and B in X, there are disjoint $N_{eu}gs\alpha^{*}$-open sets U and V in X containing A and B respectively.

Theorem 6.6. Every $N_{eu}gs\alpha^{*}$-Normal space is strongly $N_{eu}gs\alpha^{*}$-Normal.

Proof. Suppose X is $N_{eu}gs\alpha^{*}$-Normal. Let A and B be disjoint neutrosophic closed sets in X. Then A and B are $N_{eu}gs\alpha^{*}$-closed in X. Since X is $N_{eu}gs\alpha^{*}$-Normal, there exist disjoint neutrosophic open sets U and V containing A and B respectively. Since every neutrosophic open set is $N_{eu}gs\alpha^{*}$-open set. Therefore U and V are $N_{eu}gs\alpha^{*}$-open sets in X. This implies that X is strongly $N_{eu}gs\alpha^{*}$-Normal.

Theorem 6.7. Let (X,T_N) be a N_{eu}-Top-Space. Then the following statements are equivalent:

(a) X is strongly $N_{eu}gs\alpha^{*}$-Normal.

(b) For every neutrosophic closed set F in X and every neutrosophic open set U containing F, there exists a $N_{eu}gs\alpha^{*}$-open set V containing F such that $N_{eu}gs\alpha^{*}Cl(V)\subseteq U$.

(c) For each pair of disjoint neutrosophic closed sets A and B in X, there exists a $N_{eu}gs\alpha^{*}$-open set U containing A such that $N_{eu}gs\alpha^{*}Cl(U)\cap B=0_N$.

Proof. $(a)\Rightarrow(b)$: Let U be a neutrosophic open set containing neutrosophic closed set F. Then $H=U^C$ is a neutrosophic closed set disjoint from F. Since X is strongly $N_{eu}gs\alpha^{*}$-Normal, there exist disjoint $N_{eu}gs\alpha^{*}$-open sets V and W containing F and H respectively. Then $N_{eu}gs\alpha^{*}Cl(V)$ is disjoint from H, since if $y_{(r,t,s)}\in H$, the set W is a $N_{eu}gs\alpha^{*}$-open set containing $y_{(r,t,s)}$ disjoint from V. Hence $N_{eu}gs\alpha^{*}Cl(V)\subseteq U$.

$(b)\Rightarrow(c)$: Let A and B be disjoint neutrosophic closed sets in X. Then B^C is a neutrosophic open set containing A. By (b), there exists a $N_{eu}gs\alpha^{*}$-open set U containing A such that $N_{eu}gs\alpha^{*}Cl(U)\subseteq B^C$. Hence $N_{eu}gs\alpha^{*}Cl(U)\cap B=0_N$. This proves (c).

$(c)\Rightarrow(a)$: Let A and B be disjoint $N_{eu}gs\alpha^{*}$-closed sets in X. By (c), there exists a $N_{eu}gs\alpha^{*}$-open set U containing A such that $N_{eu}gs\alpha^{*}Cl(U)\cap B=0_N$. Take $V=\left(N_{eu}gs\alpha^{*}Cl(U)\right)^{C}$. Then U and V are disjoint $N_{eu}gs\alpha^{*}$-open sets containing A and B respectively. Thus X is strongly $N_{eu}gs\alpha^{*}$-Normal.

Theorem 6.8. Let (X,T_N) be a N_{eu}-Top-Space. Then the following statements are equivalent:

(a) X is strongly $N_{eu}gs\alpha^{*}$-Normal.

(b) For any two neutrosophic open sets U and V whose union is 1_N, there exist $N_{eu}gs\alpha^{*}$-closed subsets A of U and B of V such that $A\cup B=1_N$.

Proof. $(a)\Rightarrow(b)$: Let U and V be two neutrosophic open sets in a strongly $N_{eu}gs\alpha^*-Normal$ space X such that $U\bigcup V=1_N$. Then U^C and V^C are disjoint neutrosophic closed sets. Since X is strongly $N_{eu}gs\alpha^*-Normal$, then there exist disjoint $N_{eu}gs\alpha^*-open$ sets G and H such that $U^C\subseteq G$ and $V^C\subseteq H$. Let $A=G^C$ and $B=H^C$. Then A and B are $N_{eu}gs\alpha^*-closed$ subsets of U and V respectively such that $A\bigcup B=1_N$.

$(b)\Rightarrow(a)$: Let A and B be disjoint neutrosophic closed sets in X. Then A^C and B^C are neutrosophic open sets such that $A^C\bigcup B^C=1_N$. By (b), there exist $N_{eu}gs\alpha^*-closed$ sets G and H such that $G\subseteq A^C$, $H\subseteq B^C$ and $G\bigcup H=1_N$. Then G^C and H^C are disjoint $N_{eu}gs\alpha^*-open$ sets containing A and B respectively. Therefore, X is strongly $N_{eu}gs\alpha^*-Normal$.

VII. CONCLUSION

Topology is an important and major area of mathematics, and it can give many relationships between other scientific areas and mathematical models. Recently, many scientists have studied the neutrosophic set theory, which is initiated by Molodtsov and easily applied to many problems having uncertainties from social life. In the present work, we have continued to study the properties of neutrosophic topological spaces. we introduced the idea of new types of neutrosophic compactness, neutrosophic connectedness, neutrosophic regular spaces, and neutrosophic normal spaces defined in terms of neutrosophic generalized semi alpha star open sets and neutrosophic generalized semi alpha star closed sets in a neutrosophic topological space (X,T_N) namely,

$N_{eu}gs\alpha^*-compact$ spaces, $N_{eu}gs\alpha^*-Lindelof$ space, countably $N_{eu}gs\alpha^*-compact$ spaces, $N_{eu}gs\alpha^*-connected$ spaces, $N_{eu}gs\alpha^*-cseparated$ sets, $N_{eu}-Super-gs\alpha^*-connected$ spaces, $N_{eu}-Extremely-gs\alpha^*-disconnected$ spaces, and $N_{eu}-Strongly-gs\alpha^*-connected$ spaces, $N_{eu}gs\alpha^*-Regular$ spaces, strongly $N_{eu}gs\alpha^*-Regular$ spaces, $N_{eu}gs\alpha^*-Normal$ spaces, and strongly $N_{eu}gs\alpha^*-Normal$ spaces. Also, several of their topological properties are investigated. Finally, some effects of various kinds of neutrosophic functions on them are studied. and have established several interesting properties. Because there exist compact connections between neutrosophic sets and information systems, we can use the results deducted from the studies on neutrosophic topological space to improve these kinds of connections. We see that this chapter will help researcher enhance and promote the further study on neutrosophic topology to carry out a general framework for their applications in practical life.

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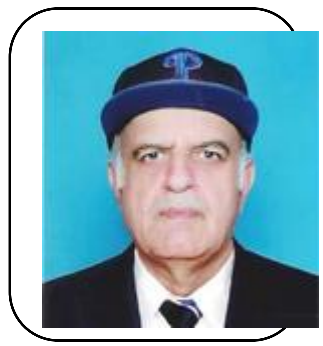

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I completed B.Sc. degree in Mathematics and Physics from the University of Punjab, Lahore, Pakistan in 1972. I completed M.Sc. in Mathematics and M. Phil in Mathematics in 1975 and 1976 respectively from the Quaid-i-Azam University, Islamabad, Pakistan. Soon after my graduation in the same year, I started teaching as a lecturer in Mathematics at the same University. In 1980 I obtained DAAD scholarship and worked as a research fellow at the University of Osnabruck, Germany, for two years. I had been teaching different graduate courses including Topology, Real Analysis, Complex variables, Functional Analysis, Group Theory, Linear Algebra, Rings and Modules, Ordinary Differential equations at the Quaid-i-Azam University, Islamabad, Pakistan. In 1985 I obtained the Canadian International/University of Alberta Scholarship and started my Ph.D. in Mathematics at the University of Alberta, Edmonton, Canada. I completed my Ph.D. in Mathematics in 1989. Then in the same year, I joined Northern Illinois University, DeKalb, U.S.A. as a teaching assistant. In 1990 I Joined the University of Southern Maine, Portland, U.S.A. with the same teaching assistant position. In 1991 I joined King Fahd University of Petroleum and Minerals, Dhahran, Saudi Arabia, as an Assistant Professor of Mathematics. I had been teaching

General Topology at the Graduate level and different courses of Mathematics including Advanced Calculus, General Topology, Calculus I, II, III, Calculus for Business Students, Ordinary Differential Equations, Finite Mathematics, Algebra, Trigonometry, Linear Algebra, Business Calculus, and Probability and Statistics at the undergraduate level. I published about thirty Technical Reports in the Department of Mathematical Sciences at KFUPM. I have given several seminars at KFUPM. I have examined at least half a dozen M.S. Dissertations in Topology at different universities in Saudi Arabia as an external examiner. I joined Prince Mohammad bin Fahd University as an Assistant Professor of Mathematics in 2011. I had made contributions to several dozens of committees and technical groups consecutively at KFUPM and PMU. I have participated in more than two hundred national and international workshops at KFUPM and PMU. I also participated and made presentations in about fifty national and international conferences. I have taught different courses in Mathematics including Linear Algebra, Statistical Methods, Introductory Algebra, Calculus for Business Students, Finite Mathematics, Calculus – I, II, III, Ordinary Differential Equations, Probability and Statistics, Linear Algebra and Differential Equations. During my career of twenty years at KFUPM and ten years at PMU, I have published over one hundred research papers in International Journals in the fields of General Topology, Fuzzy Topology, Neutrosophic Topology, Topological Vector Spaces, Algebra, Statistics. and Mathematics Education.

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