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Neutrosophic grb-Continuous and grb-Irresolute Mappings in Neutrosophic Topological Spaces

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***Abstract*— Real-life structures always include indeterminacy. The Mathematical tool which is well known in dealing with indeterminacy is neutrosophic. Smarandache proposed the approach of neutrosophic sets. Neutrosophic sets deal with uncertain data. The notion of neutrosophic set is generally referred to as the generalization of intuitionistic fuzzy set. In 2021, Dr. G. Sindhu introduced the concept of Neutrosophic generalized regular b-closed sets and neutrosophic generalized b-open sets and presented some of their properties in Neutrosophic topological spaces. In this research paper, we introduce the concepts of neutrosophic** grb−continuous **mappings, neutrosophic** grb−irresolute **mappings, neutrosophic** grb−closed **mappings, neutrosophic** grb−open **mappings, strongly neutrosophic** grb−continuous **mappings, perfectly neutrosophic** grb−continuous **mappings, neutrosophic contra** grb−continuous **mappings and neutrosophic contra** grb−irresolute **mappings in neutrosophic topological spaces. We investigate and obtain several properties and characterizations concerning these mappings in neutrosophic topological spaces.**

***Keywords*: Neutrosophic topological space; Neutrosophic** grb−open **set; Neutrosophic** grb−closed **set; Neutrosophic** grb−continuous **mapping; Neutrosophic** grb−irresolute **mapping; Neutrosophic** grb−open **mapping; Neutrosophic** grb−closed **mapping; Strongly neutrosophic** grb−continuous **mapping; Perfectly neutrosophic** grb−continuous **mapping; Neutrosophic contra** grb−continuous **mapping; Neutrosophic contra** grb−irresolute **mapping**

I.INTRODUCTION

Many real-life problems in Business, Finance, Medical Sciences, Engineering, and Social Sciences deal with uncertainties. Smarandache studies neutrosophic set as an approach for solving issues that cover unreliable, indeterminacy, and persistent data. Applications of neutrosophic topology depend upon the properties of neutrosophic closed sets, neutrosophic open sets, neutrosophic interior operator, neutrosophic closure operator, and neutrosophic sets. In 2021, Dr. G. Sindhu introduced the concepts of Neutrosophic generalized regular b-closed sets and Neutrosophic generalized b-open sets and presented some some of their properties in Neutrosophic topological spaces. We introduce the concepts of neutrosophic grb−continuous mappings, neutrosophic grb−irresolute mappings, neutrosophic grb−closed mappings, neutrosophic grb−open mappings, strongly neutrosophic grb−continuous mappings, perfectly neutrosophic grb−continuous mappings, neutrosophic contra grb−continuous mappings and neutrosophic contra grb−irresolute mappings in neutrosophic topological spaces. We investigate and obtain several properties and characterizations concerning these mappings in neutrosophic topological spaces.

II. PRELIMINARIES

Definition 2.1. Let X be a non-empty fixed set. A neutrosophic set P is an object having the form $P=\left\{\left\langle x,\mu_P(x),\sigma_P(x),\gamma_P(x)\right\rangle : x\in X\right\}$, where $\mu_P(x)$- represents the degree of membership, $\sigma_P(x)$- represents the degree of indeterminacy, and $\gamma_P(x)$- represents the degree of non-membership.

Definition 2.2. A neutrosophic topology on a non-empty set X is a family T_N of neutrosophic subsets of X satisfying (i) $0_N, 1_N \in T_N$. (ii) $G \cap H \in T_N$ for every $G, H \in T_N$, (iii) $\bigcup_{j \in J} G_j \in T_N$ for every $\{G_j : j \in J\} \subseteq \tau_N$.

Then the pair (X, T_N) is called a neutrosophic topological space. The elements of T_N are called neutrosophic open sets in X. A neutrosophic set A is called a neutrosophic closed set if and only if its complement A^C is a neutrosophic open set.

Definition 2.3. Let (X, T_N) be a neutrosophic topological space and A be a neutrosophic set. Then

(i) The neutrosophic interior of A, denoted by $N_{eu}Int(A)$ is the union of all neutrosophic open subsets of X contained in A.

(ii) The neutrosophic closure of A denoted by $N_{eu}Cl(A)$ is the intersection of all neutrosophic closed sets containing A.

Definition 2.4. Let A be a neutrosophic set in a neutrosophic topological space (X, T_N). Then the set A is called a neutrosophic regular open set in a neutrosophic topological space X if $A \subseteq N_{eu}Int[N_{eu}Cl(A)]$.

Definition 2.5. Let A be a neutrosophic set in a neutrosophic topological space (X, T_N). Then the set A is called a neutrosophic α-open set in neutrosophic topological space X if $A \subseteq N_{eu}Int[N_{eu}Cl(N_{eu}In(A))]$.

Definition 2.6. Let A be a neutrosophic set in a neutrosophic topological space (X, T_N). Then the set A is called a neutrosophic b-open set in neutrosophic topological space (X, T_N) if $A \subseteq N_{eu}Int[N_{eu}Cl(A)] \cup N_{eu}Cl[N_{eu}Int(A)]$.

Definition 2.7. Let A be a neutrosophic set in a neutrosophic topological space (X, T_N). Then the set A is called a Neutrosophic Generalized Regular b-closed (briefly $\mathcal{g}$rb-closed) set in neutrosophic topological space (X, T_N) if $N_{eu}bCl(A) \subseteq U$ whenever $A \subseteq U$ and U is neutrosophic regular open set in X.

Definition 2.8. Let A be a neutrosophic set in a neutrosophic topological (X, T_N). Then the set A is called a Neutrosophic Generalized Regular b-open (briefly $\mathcal{g}$rb-open) set in neutrosophic topological (X, T_N) if the complement A^C of A is neutrosophic $\mathcal{grb}$-closed set in X.

Definition 2.9. Let A be a subset of a neutrosophic topological (X, T_N). Then neutrosophic generalized regular b-interior of A is given by: $N_{eu}\mathcal{g}rbInt(A) = \bigcup\{G : G \text{ is a } N_{eu}\mathcal{grb}\text{-open set in } X \text{ and } G \subseteq A\}$.

Definition 2.10. Let A be a subset of a neutrosophic topological (X, T_N). Then neutrosophic generalized regular b-closure of A is $N_{eu}\mathcal{g}rbCl(A) = \bigcap\left\{\begin{array}{l} G : G \text{ is a neutrosophic } \mathcal{g}rb\text{-closed set in } X \\ \text{and } A \subseteq G \end{array}\right\}$.

Remark 2.11. Let A be a subset of a neutrosophic topological (X, T_N). Then $N_{eu}\mathcal{g}rbInt(A)$ is neutrosophic $\mathcal{g}$rb-open set in (X, T_N). The complement of $N_{eu}\mathcal{g}rbInt(A)$ is $N_{eu}\mathcal{grb}Cl(A)$.

Theorem 2.12. Every neutrosophic closed (resp. open) set in a neutrosophic topological space is neutrosophic $\mathcal{grb}$-closed (resp. neutrosophic $\mathcal{grb}$-open) set.

Theorem 2.13. Every neutrosophic α-closed set in a neutrosophic topological space (X, T_N) is neutrosophic $\mathcal{grb}$-closed set.

Theorem 2.14. The union of any two neutrosophic $\mathcal{grb}$-closed sets in a neutrosophic topological space (X, T_N) is also a neutrosophic $\mathcal{grb}$-closed set in (X, T_N).

Theorem 2.15. The intersection of any two neutrosophic $\mathcal{grb}$-open sets in a neutrosophic topological space (X, T_N) is also a neutrosophic $\mathcal{grb}$-open set in (X, T_N).

Theorem 2.16. The union of any family of neutrosophic grb–open sets in a neutrosophic topological space (X, T_N) is also a neutrosophic grb–open set in (X, T_N).

Definition 2.17. Let A be a neutrosophic subset of a neutrosophic topological space (X, T_N). Then the neutrosophic grb–frontier of a neutrosophic subset A of X is denoted by $N_{eu}grbFr(A)$ and is defined by $N_{eu}grbFr(A) = N_{eu}grbCl(A) \cap N_{eu}grbCl(A^C)$.

Theorem 2.18. For a neutrosophic set A in a neutrosophic topological space (X, T_N), the following statements are true:

(i) $\left[N_{eu}grbInt(A)\right]^C = N_{eu}grbCl(A^C)$.

(ii) $\left[N_{eu}grbCl(A)\right]^C = N_{eu}grbInt(A^C)$.

Definition 2.19. Let $f:(X, T_N) \to (Y, \sigma_N)$ be a mapping. Then f is called a neutrosophic continuous mapping if $f^{-1}(V)$ is a neutrosophic open set in X for every neutrosophic open set V in Y.

Theorem 2.20. Let $f:(X, T_N) \to (Y, \sigma_N)$ be a mapping. Then f is called a neutrosophic continuous mapping if $f^{-1}(V)$ is a neutrosophic closed set in X for every neutrosophic closed set V in Y.

III. NEUTROSOPHIC grb-CONTINUOUS MAPPINGS

In this section, we introduce the concepts of neutrosophic grb–continuous mappings in neutrosophic topological spaces. Also, we study some of the main results depending on neutrosophic grb–open sets.

Definition 3.1. Let $f:(X, T_N) \to (Y, \sigma_N)$ be a mapping. Then f is called a neutrosophic grb–continuous mapping if $f^{-1}(V)$ is a neutrosophic grb–open set in X for every neutrosophic open set V in Y.

Theorem 3.2. Every neutrosophic continuous mapping is neutrosophic grb–continuous mapping.

Proof. Let $f:(X, T_N) \to (Y, \sigma_N)$ be neutrosophic continuous mapping. Let V be a neutrosophic open set in (Y, σ_N). Then $f^{-1}(V)$ is neutrosophic open set in (X, T_N). Since every neutrosophic open set is neutrosophic grb–open, $f^{-1}(V)$ is neutrosophic grb–open set in (X, T_N). Hence f is neutrosophic grb–continuous mapping.

Theorem 3.3. Let (X, T_N), (Y, σ_N) and (Z, η_N) be neutrosophic topological spaces. If $f:(X, T_N) \to (Y, \sigma_N)$ is a neutrosophic grb–continuous mapping and $g:(Y, \sigma_N) \to (Z, \eta_N)$ is neutrosophic grb–continuous, then $gof:(X, T_N) \to (Z, \eta_N)$ is a is neutrosophic grb–continuous mapping.

Proof. Let G be a neutrosophic open set in Z. Since $g:(Y, \sigma_N) \to (Z, \eta_N)$ is neutrosophic continuous, $f^{-1}(G)$ is neutrosophic open in Y. Since f is a neutrosophic grb–continuous mapping, $f^{-1}\left[f^{-1}(G)\right]$ is neutrosophic grb–open in X. But $f^{-1}\left[g^{-1}(G)\right] = (gof)^{-1}(G)$. Then $(gof)^{-1}(G)$ is neutrosophic grb–open set in X. Hence, gof is a neutrosophic grb–continuous mapping.

Theorem 3.4. Let (X, T_N) and (Y, σ_N) be two neutrosophic topological spaces. Then prove that $f:(X, T_N) \to (Y, \sigma_N)$ is neutrosophic grb–continuous if and only if $f^{-1}(B$ is neutrosophic grb–closed set in X for every neutrosophic closed set B in Y.

Proof. Let B be a neutrosophic closed set in Y. Then B^C is neutrosophic open set in Y. Since f is neutrosophic grb–continuous. Therefore $f^{-1}(B^C)$ is a neutrosophic grb–open set in X. Since $f^{-1}(B^C) = \left[f^{-1}(B)\right]^C$, $f^{-1}(B)$ is neutrosophic grb–closed set in X.

Conversely, Let B be a neutrosophic open set in Y. Then B^C is neutrosophic closed set in Y. By assumption $f^{-1}(B^C)$ is neutrosophic $grb-$closed set in X. Since $f^{-1}(B^C)=\left[f^{-1}(B)\right]^C$, $f^{-1}(B)$ is neutrosophic $grb-$open set in X. Hence f is neutrosophic $grb-$continuous.

Theorem 3.5. Let (X,T_N) and (Y,σ_N) be two neutrosophic topological spaces and $f:X\to Y$ be a mapping. Then f is a neutrosophic $grb-$continuous mapping if and only if $f\left(N_{eu}grbCl(A)\right)\subseteq N_{eu}grbCl\left(f(A)\right)$ for every neutrosophic set A in X.

Proof. Let A be a neutrosophic set in X and f be a neutrosophic $grb-$continuous mapping. Then evidently $f(A)\subseteq N_{eu}grbCl\left[f(A)\right]$. Now, $A\subseteq f^{-1}\left[f(A)\right]\subseteq f^{-1}\left[N_{eu}grbCl\left(f(A)\right)\right]$ and $N_{eu}grbCl(A)\subseteq$ $N_{eu}grbCl\left[f^{-1}\left(N_{eu}grbCl\left(f(A)\right)\right)\right]$. Since f is a neutrosophic $N_{eu}grb-$continuous mapping and $N_{eu}grbCl\left[f(A)\right]$ is a neutrosophic $grb-$closed set. Thus $N_{eu}grbCl\left[f^{-1}\left(N_{eu}grbCl\left(f(A)\right)\right)\right]=$ $f^{-1}\left[N_{eu}grbCl\left(f(A)\right)\right]$. Hence, $f\left[N_{eu}grbCl(A)\right]\subseteq N_{eu}grbCl\left[f(A)\right]$.

Conversely, let $f\left[N_{eu}grbCl(A)\right]\subseteq N_{eu}grbCl\left[f(A)\right]$, for each neutrosophic set A in X. Let F be a neutrosophic closed set in Y. Then $N_{eu}grbCl\left[f\left(f^{-1}(F)\right)\right]\subseteq N_{eu}grbCl(F)=F$. By assumption, $f\left[N_{eu}grbCl\left(f^{-1}(F)\right)\right]\subseteq N_{eu}grbCl\left[f\left(f^{-1}(F)\right)\right]$ $\subseteq F$ and hence $N_{eu}grbCl\left[f^{-1}(F)\right]\subseteq f^{-1}(F)$. Since $f^{-1}(F)\subseteq N_{eu}grbCl\left[f^{-1}(F)\right]$, $N_{eu}grbCl\left[f^{-1}(F)\right]=f^{-1}(F)$. This implies that $f^{-1}(F)$ is a neutrosophic $grb-$closed set in X. Thus by Theorem 3.4, f is a neutrosophic $grb-$continuous mapping.

Theorem 3.6. Let (X,T_N) and (Y,σ_N) be two neutrosophic topological spaces and $f:X\to Y$ be a mapping. Then f is a neutrosophic $grb-$continuous mapping if and only if $N_{eu}grbCl\left[f^{-1}(B)\right]\subseteq f^{-1}\left[N_{eu}grbCl(B)\right]$ for every neutrosophic set B in Y.

Proof. Let B be any neutrosophic set in Y and f be a neutrosophic $grb-$continuous mapping. Clearly $f^{-1}(B)\subseteq f^{-1}\left[N_{eu}grbCl(B)\right]$. Then, $N_{eu}grbCl\left[f^{-1}(B)\right]\subseteq N_{eu}grbCl\left[f^{-1}\left(N_{eu}grbCl(B)\right)\right]$. Since $N_{eu}grbCl(B)$ is neutrosophic $grb-$closed set in Y. So by Theorem 3.4, $f^{-1}\left[N_{eu}grbCl(B)\right]$ is a neutrosophic $grb-$closed set in X. Thus, $N_{eu}grbCl\left[f^{-1}(B)\right]\subseteq$ $N_{eu}grbCl\left[f^{-1}\left(N_{eu}grbCl(B)\right)\right]=f^{-1}\left[N_{eu}grbCl(B)\right]$.

Conversely, $N_{eu}grbCl\left[f^{-1}(B)\right]\subseteq f^{-1}\left[N_{eu}grbCl(B)\right]$ for all neutrosophic sets B in Y. Let F be a neutrosophic closed set in Y. Since every neutrosophic closed set is neutrosophic $grb-$closed set, $N_{eu}grbCl\left[f^{-1}(F)\right]\subseteq f^{-1}\left[N_{eu}grbCl(F)\right]=f^{-1}(F)$. This implies that $f^{-1}(F)$ is a neutrosophic $grb-$closed set in X. Thus by Theorem 3.4, f is a neutrosophic $grb-$continuous mapping.

Theorem 3.7. Let (X,T_N) and (Y,σ_N) be two neutrosophic topological spaces and $f:X\to Y$ be a bijective mapping. Then f is neutrosophic $grb-$continuous if and only if $N_{eu}grbInt\left[f(A)\right]\subseteq f\left[N_{eu}grbInt(A)\right]$ for every neutrosophic set A in X.

Proof. Let A be any neutrosophic set in X and f be a bijective and neutrosophic $grb-$continuous mapping. Let $f(A)=B$. Clearly

$f^{-1}\left[N_{eu}grbInt(B)\right]\subseteq f^{-1}(B)$. Since f is an injective mapping, $f^{-1}(B)=A$, so that $f^{-1}\left[N_{eu}grbInt(B)\right]\subseteq A$. Therefore,

$N_{eu}grbInt\left[f^{-1}\left(N_{eu}grbInt(B)\right)\right]\subseteq N_{eu}grbInt(A)$.

Since f is neutrosophic grb-continuous, $f^{-1}\left[N_{eu}grbInt(B)\right]$ is neutrosophic grb-open set in X and

$f^{-1}\left[N_{eu}grbInt(B)\right]\subseteq N_{eu}grbInt(A)$,

$f\left[f^{-1}\left(N_{eu}grbInt(B)\right)\right]\subseteq f\left[N_{eu}grbInt(A)\right]$.

Thus we obtain $N_{eu}grbInt\left[f(A)\right]\subseteq f\left[N_{eu}grbInt(A)\right]$.

Conversely, $N_{eu}grbInt\left[f(A)\right]\subseteq f\left[N_{eu}grbInt(A)\right]$ for every neutrosophic set A in X. Let V be a neutrosophic open set in Y. Then V is neutrosophic grb-open set in Y. Since f is surjective and so

$V=N_{eu}grbInt(V)=N_{eu}grbInt\left[f\left(f^{-1}(V)\right)\right]$

$\subseteq f\left[N_{eu}grbInt\left(f^{-1}(V)\right)\right]$. It follows that $f^{-1}(V)\subseteq N_{eu}grbInt\left[f^{-1}(V)\right]$.

Therefore $f^{-1}(V)$ is neutrosophic grb-open set in X. Hence f is a neutrosophic grb-continuous mapping.

Theorem 3.8. Let (X,T_N) and (Y,σ_N) be two neutrosophic topological spaces and $f:X\to Y$ be a mapping. Then f is a neutrosophic grb-continuous mapping if and only if $f^{-1}\left[N_{eu}grbInt(B)\right]\subseteq N_{eu}grbInt\left[f^{-1}(B)\right]$ for every neutrosophic set B in Y.

Proof. Let B be any neutrosophic set in Y and f be a neutrosophic grb-continuous mapping. Clearly $f^{-1}\left[N_{eu}grbInt(B)\right]\subseteq f^{-1}(B)$ implies

$N_{eu}grbInt\left[f^{-1}\left(N_{eu}grbInt(B)\right)\right]\subseteq$

$N_{eu}grbInt\left[f^{-1}(B)\right]$.

Since $N_{eu}grbInt(B)$ is neutrosophic grb-open set in Y and f is neutrosophic grb-continuous, $f^{-1}\left[N_{eu}grbInt(B)\right]$ is neutrosophic grb-open set in X. Thus

$N_{eu}grbInt\left[f^{-1}\left(N_{eu}grbInt(B)\right)\right]\subseteq$

$f^{-1}\left[N_{eu}grbInt(B)\right]\subseteq N_{eu}grbInt\left[f^{-1}(B)\right]$.

Conversely, $f^{-1}\left[N_{eu}grbInt(B)\right]\subseteq N_{eu}grbInt\left[f^{-1}(B)\right]$ for every neutrosophic set B in Y. Let G be any neutrosophic open set in Y. Then $f^{-1}(G)=f^{-1}\left[N_{eu}grbInt(G)\right]\subseteq N_{eu}grbInt\left[f^{-1}(G)\right]$ and therefore $f^{-1}(G)=N_{eu}grbInt\left[f^{-1}(G)\right]$. This implies that $f^{-1}(G)$ is neutrosophic grb-open set in X. Hence f is a neutrosophic grb-continuous mapping.

Theorem 3.9. Let (X,T_N) and (Y,σ_N) be two neutrosophic topological spaces and $f:X\to Y$ be a bijective mapping. Then f is a neutrosophic grb-continuous mapping if and only if $f\left[N_{eu}grbFr(A)\right]\subseteq N_{eu}grbFr\left[f(A)\right]$ for every neutrosophic set A in X.

Proof. Let f be a bijective and neutrosophic grb-continuous mapping. Let A be a neutrosophic set in X. By definition, $N_{eu}grbFr(A)=N_{eu}grbCl(A)\cap N_{eu}grbCl\left(A^C\right)$.

By Theorem 3.7, $N_{eu}grbInt\left[f(A)\right]\subseteq f\left[N_{eu}grbInt(A)\right]$ and from Theorem 3.5,

$f\left[N_{eu}grbCl(A)\right]\subseteq N_{eu}grbCl\left[f(A)\right]$,

$f\left[N_{eu}grbFr(A)\right]=$

$f\left[N_{eu}grbCl(A)\right]\cap f\left[N_{eu}grbCl\left(A^C\right)\right]\subseteq$

$N_{eu}grbCl\left[f(A)\right]\cap N_{eu}grbCl\left[f(A)\right]^C$

$=N_{eu}grbFr\left[f(A)\right]$.

Conversely, $f\left[N_{eu}grbFr(A)\right]\subseteq N_{eu}grbFr\left[f(A)\right]$ for every neutrosophic set A in X. Then

$f\left[N_{eu}grbCl(A)\right]=f\left[N_{eu}grbInt(A)\right]\cup f\left[N_{eu}grbFr(A)\right]$

$\subseteq f(A)\cup N_{eu}grbFr\left[f(A)\right]\subseteq N_{eu}grbCl\left[f(A)\right]$.

By Theorem 3.5, f is a neutrosophic $grb-$continuous mapping.

Theorem 3.10. Let (X, T_N) and (Y, σ_N) be two neutrosophic topological spaces and $f: X \to Y$ be a bijective mapping. Then f is a neutrosophic $grb-$continuous mapping if and only if $N_{eu}grbFr[f^{-1}(B)] \subseteq f^{-1}[N_{eu}grbFr(B)]$ for every neutrosophic set B in Y.

Proof. Let f be a bijective and neutrosophic $grb-$continuous mapping. Let B be a neutrosophic set in Y. By Theorem 3.6, $N_{eu}grbCl[f^{-1}(B)] \subseteq f^{-1}[N_{eu}grbCl(B)]$. So $f^{-1}[N_{eu}grbFr(B)] =$
$f^{-1}[(N_{eu}grbCl(B)) \cap N_{eu}grbCl(B^C)] =$
$f^{-1}[N_{eu}grbCl(B)] \cap f^{-1}[N_{eu}grbCl(B^C)] \supseteq$
$N_{eu}grbCl[f^{-1}(B)] \cap N_{eu}grbCl[f^{-1}(B^C)] =$
$N_{eu}grbCl[f^{-1}(B)] \cap N_{eu}grbCl[(f^{-1}(B))^C]$
$= N_{eu}grbFr[f^{-1}(B)]$. Therefore $N_{eu}grbFr[f^{-1}(B)] \subseteq f^{-1}[N_{eu}grbFr(B)]$.

Conversely since $N_{eu}grbFr[f^{-1}(B)] \subseteq f^{-1}[N_{eu}grbFr(B)]$ for every neutrosophic set B in Y. This implies that $N_{eu}grbCl[f^{-1}(B)] \subseteq f^{-1}[N_{eu}grbCl(B)]$. By Theorem 3.6, f is a neutrosophic $grb-$continuous mapping.

Definition 3.11. Let $x_{(r,t,s)}$ be a neutrosophic point of a neutrosophic topological space (X, T_N). A neutrosophic set A of X is called neutrosophic neighbourhood of $x_{(r,t,s)}$ if there exists a neutrosophic open set B such that $x_{(r,t,s)} \in B \subseteq A$.

Theorem 3.12. Let f be a mapping from a neutrosophic topological space (X, T_N) to a neutrosophic topological space (Y, σ_N). Then the following assertions are equivalent.

(i) f is neutrosophic $grb-$continuous.

(ii) For each neutrosophic point $x_{(r,t,s)} \in X$ and every neutrosophic neighbourhood A of $f(x_{(r,t,s)})$, there exists a neutrosophic $grb-$open set B such that $x_{(r,t,s)} \in B \subseteq f^{-1}(A)$.

(iii) For each neutrosophic point $x_{(r,t,s)} \in X$ and every neutrosophic neighbourhood A of $f(x_{(r,t,s)})$, there exists a neutrosophic $grb-$open set B in X such that $x_{(r,t,s)} \in B$ and $f(B) \subseteq A$.

Proof. $(\text{i}) \Rightarrow (\text{ii})$: Let $x_{(r,t,s)} \in X$ be a neutrosophic point in X and let A be a neutrosophic neighbourhood of $f(x_{(r,t,s)})$. Then there exists a neutrosophic open set B in Y such that $f(x_{(r,t,s)}) \in B \subseteq A$. Since f is neutrosophic $grb-$continuous, we know that $f^{-1}(B)$ is a neutrosophic $grb-$open set in X and $x_{(r,t,s)} \in f^{-1}(f_{(r,t,s)}) \subseteq f^{-1}(B) \subseteq f^{-1}(A)$. This implies (ii) is true.

$(\text{ii}) \Rightarrow (\text{iii})$: Let $x_{(r,t,s)}$ be a neutrosophic point in X and let A be a neutrosophic neighbourhood of $f(x_{(r,t,s)})$. The condition (ii) implies that there exists a neutrosophic $grb-$open set B in X such that $x_{(r,t,s)} \in B \subseteq f^{-1}(A)$. Thus $x_{(r,t,s)} \in B$ and $f(B) \subseteq f[f^{-1}(A)] \subseteq A$. Hence (iii) is true.

$(\text{iii}) \Rightarrow (\text{i})$: Let B be a neutrosophic open set in Y and let $x_{(r,t,s)} \in f^{-1}(B)$. Since B is neutrosophic open set, $f(x_{(r,t,s)}) \in B$, and so B is neutrosophic neighbourhood of $f(x_{(r,t,s)})$. It follows from (iii) that there exists a neutrosophic $grb-$open set A in X such that $x_{(r,t,s)} \in A$ and $f(A) \subseteq B$ so that $x_{(r,t,s)} \in A \subseteq f^{-1}[f(A)] \subseteq f^{-1}(B)$. This implies by definition that $f^{-1}(B)$ is a neutrosophic $grb-$open set in X. Therefore, f is a neutrosophic $grb-$continuous mapping.

IV. NEUTROSOPHIC grb-IRRESOLUTE MAPPINGS

In this section, we introduce the concept of neutrosophic $grb-$irresolute mappings in neutrosophic topological spaces. Also, we discuss the relationship with neutrosophic $grb-$continuous mappings.

Definition 4.1. Let (X,T_N) and (Y,σ_N) be two neutrosophic topological spaces. A mapping $f: X \to Y$ is called neutrosophic $grb-$irresolute if the inverse image of every neutrosophic $grb-$open set in Y is neutrosophic $grb-$open in X.

Theorem 4.2. Let (X,T_N) and (Y,σ_N) be two neutrosophic topological spaces. A mapping $f: X \to Y$ is called neutrosophic $grb-$irresolute if the inverse image of every neutrosophic $grb-$closed set in Y is neutrosophic $grb-$closed in X.

Proof. Let A be any neutrosophic $grb-$closed set in Y. Then A^C is neutrosophic $grb-$open set in Y. Since f is neutrosophic $grb-$irresolute, $f^{-1}(A^C)$ is neutrosophic $grb-$open set in X and $f^{-1}(A^C)=\left[f^{-1}(A)\right]^C$ which implies that $f^{-1}(A)$ is neutrosophic $grb-$closed set in X.

Conversely, Let B be any neutrosophic $grb-$open set in Y. Then B^C is neutrosophic $grb-$closed set in Y. Thus $f^{-1}(B^C)$ is neutrosophic $grb-$closed set in X and $f^{-1}(B^C)=\left[f^{-1}(B)\right]^C$ which implies that $f^{-1}(B)$ is neutrosophic $grb-$open set in X. Hence $f: X \to Y$ is neutrosophic $grb-$irresolute.

Theorem 4.3. Every neutrosophic $grb-$irresolute mapping is neutrosophic $grb-$continuous.

Proof. Let V be a neutrosophic open set in Y. Since every neutrosophic open set is neutrosophic $grb-$open, V is neutrosophic $grb-$open. Since f is neutrosophic $grb-$irresolute, $f^{-1}(V)$ is neutrosophic $grb-$open in X. Therefore f is neutrosophic $grb-$continuous.

Theorem 4.4. Let $f:(X,T_N)\to(Y,\sigma_N)$ be a mapping. Then the following assertions are equivalent:

(i) f is neutrosophic $grb-$irresolute.

(ii) $N_{eu}grbCl\left[f^{-1}(B)\right]\subseteq f^{-1}\left[N_{eu}grbCl(B)\right]$ for every neutrosophic set B of Y.

(iii) $f\left[N_{eu}grbCl(A)\right]\subseteq N_{eu}grbCl\left[f(A)\right]$ for every neutrosophic set A of X.

(iv) $f^{-1}\left[N_{eu}grbInt(B)\right]\subseteq N_{eu}grbInt\left[f^{-1}(B)\right]$ for every neutrosophic set B of Y.

Proof. $(i)\Rightarrow(ii)$: Let B be any neutrosophic set in Y. Then $N_{eu}grbCl(B)$ is neutrosophic $grb-$closed set in Y. Since f is neutrosophic $grb-$irresolute, $f^{-1}\left[N_{eu}grbCl(B)\right]$ is neutrosophic $grb-$closed set in X. Then $N_{eu}grbCl\left[f^{-1}(N_{eu}grbCl(B))\right]=f^{-1}\left[N_{eu}grbCl(B)\right]$. Clearly it follows that $N_{eu}grbCl\left[f^{-1}(B)\right]\subseteq N_{eu}grbCl\left[f^{-1}(N_{eu}grbCl(B))\right]=f^{-1}\left[N_{eu}grbCl(B)\right]$. This proves (ii).

$(ii)\Rightarrow(iii)$: Let A be any neutrosophic set in X. Then $f(A)\subseteq Y$. By (ii), $N_{eu}grbCl\left[f^{-1}(f(A))\right]\subseteq f^{-1}\left[N_{eu}grbCl(f(A))\right]$. But $N_{eu}grbCl(A)\subseteq N_{eu}grbCl\left[f^{-1}(f(A))\right]$, so we obtain $N_{eu}grbCl(A)\subseteq f^{-1}\left[N_{eu}grbCl(f(A))\right]$. Thus $f\left[N_{eu}grbCl(A)\right]\subseteq N_{eu}grbCl\left[f(A)\right]$.

$(iii)\Rightarrow(i)$: Let F be any neutrosophic $grb-$closed set in Y. Then $f^{-1}(F)=f^{-1}\left[N_{eu}grbCl(F)\right]$. By (iii), $f\left[N_{eu}grbCl(f^{-1}(F))\right]\subseteq N_{eu}grbCl\left[f(f^{-1}(F))\right]$ $\subseteq N_{eu}grbCl(F)=F$.

That implies, $N_{eu}grbCl\left[f^{-1}(F)\right]\subseteq f^{-1}(F)$. But $f^{-1}(F)\subseteq N_{eu}grbCl\left[f^{-1}(F)\right]$, $N_{eu}grbCl\left[f^{-1}(F)\right]=f^{-1}(F)$ and so $f^{-1}(F)$ is neutrosophic $grb-$closed set in X. Therefore f is neutrosophic $grb-$irresolute.

$(\mathrm{i}) \Rightarrow (\mathrm{iv})$: Let B be any neutrosophic set in Y. We know that $N_{eu}grbInt(B)$ is neutrosophic $grb-$open set in Y. Since f is neutrosophic $grb-$irresolute, $f^{-1}[N_{eu}grbInt(B)]$ is neutrosophic $grb-$open set in X. Then $f^{-1}[N_{eu}grbInt(B)] = N_{eu}grbInt[f^{-1}(N_{eu}grbInt(B))] \subseteq N_{eu}grbInt[f^{-1}(B)]$.

$(\mathrm{iv}) \Rightarrow (\mathrm{i})$: Let V be any neutrosophic $grb-$open set in Y. Then by (iv), $f^{-1}(V) = f^{-1}[N_{eu}grbInt(V)] \subseteq N_{eu}grbInt[f^{-1}(V)]$. But, $N_{eu}grbInt[f^{-1}(V)] \subseteq f^{-1}(V)$, $N_{eu}grbInt[f^{-1}(V)] = f^{-1}(V)$ and hence $f^{-1}(V)$ is neutrosophic $grb-$open. Thus f is neutrosophic $grb-$irresolute.

Theorem 4.5. If $f:(X,T_N) \to (Y,\sigma_N)$ and $g:(Y,\sigma_N) \to (Z,\eta_N)$ are neutrosophic $grb-$irresolute, then their composition $gof:(X,T_N) \to (Z,\eta_N)$ is also neutrosophic $grb-$irresolute.

Proof. Let V be a neutrosophic $grb-$open set in Z. Since g is a neutrosophic $grb-$irresolute mapping, $g^{-1}(V)$ is neutrosophic $grb-$open in Y. Since f is a neutrosophic $grb-$irresolute mapping, $f^{-1}[g^{-1}(V)] = (gof)^{-1}(V)$ is neutrosophic $grb-$open in X. Therefore gof is neutrosophic $grb-$irresolute.

Theorem 4.6. If $f:(X,T_N) \to (Y,\sigma_N)$ is neutrosophic $grb-$irresolute and $g:(Y,\sigma_N) \to (Z,\eta_N)$ is neutrosophic $grb-$continuous, then their composition $gof:(X,T_N) \to (Z,\eta_N)$ is also neutrosophic $grb-$continuous.

Proof. Let V be a neutrosophic open set in Z. Since g is a neutrosophic $grb-$continuous mapping, $g^{-1}(V)$ is neutrosophic $grb-$open set in Y. Since f is a neutrosophic $grb-$irresolute mapping, $f^{-1}[g^{-1}(V)] = (gof)^{-1}(V)$ is neutrosophic $grb-$open in X. Therefore gof is neutrosophic $grb-$conrinuous.

V. NEUTROSOPHIC grb-CLOSED MAPPINGS AND NEUTROSOPHIC grb-OPEN MAPPINGS

In this section, we introduce neutrosophic $grb-$closed mappings and neutrosophic $grb-$open mappings in neutrosophic topological spaces and obtain certain characterizations of these classes of mappings.

Definition 5.1. Let (X,T_N) and (Y,σ_N) be two neutrosophic topological spaces. A function $f:(X,T_N) \to (Y,\sigma_N)$ is said to be neutrosophic $grb-$closed if the image of each neutrosophic closed set in X is neutrosophic $grb-$closed in Y.

Definition 5.2. Let (X,T_N) and (Y,σ_N) be two neutrosophic topological spaces. A function $f:(X,T_N) \to (Y,\sigma_N)$ is said to be neutrosophic $grb-$open if the image of each neutrosophic open set in X is neutrosophic $grb-$open in Y.

Theorem 5.3. A function $f:(X,T_N) \to (Y,\sigma_N)$ is said to be neutrosophic $grb-$closed if and only if $N_{eu}grbCl[f(A)] \subseteq f[N_{eu}Cl(A)]$ for every neutrosophic set A of X.

Proof. Suppose $f:(X,T_N) \to (Y,\sigma_N)$ is a neutrosophic $grb-$closed function and A is any neutrosophic set in X. Then $N_{eu}Cl(A)$ is a neutrosophic closed set in X. Since f is neutrosophic $grb-$closed, $f[N_{eu}Cl(A)]$ is a neutrosophic $grb-$closed set in Y. Thus $N_{eu}grbCl[f(N_{eu}Cl(A))] = f[N_{eu}Cl(A)]$. Therefore $N_{eu}grbCl[f(A)] \subseteq N_{eu}grbCl[f(N_{eu}Cl(A))] = f(N_{eu}Cl(A))$. Hence $N_{eu}grbCl[f(A)] \subseteq f(N_{eu}Cl(A))$.

Conversely, let A be a neutrosophic closed set in X. Then $N_{eu}Cl(A)=A$ and so $f(A)=f[N_{eu}Cl(A)]$. By our assumption $N_{eu}grbCl[f(A)]\subseteq f(A)$. But $f(A)\subseteq N_{eu}grbCl[f(A)]$. Hence $N_{eu}grbCl[f(A)]=f(A)$ and therefore $f(A)$ is neutrosophic grb−closed set in Y. Thus f is a neutrosophic grb−closed mapping.

Theorem 5.4.. A mapping $f:(X,T_N)\to(Y,\sigma_N)$ is neutrosophic grb−closed if and only if for each neutrosophic set W of Y and for each neutrosophic open set U of X containing $f^{-1}(W)$ there exists a neutrosophic grb−open set V of Y such that $W\subseteq V$ and $f^{-1}(V)\subseteq U$.

Proof. Suppose f is a neutrosophic grb−closed mapping. Let W be any neutrosophic set in Y and U be a neutrosophic grb−open set of X such tha $f^{-1}(W)\subseteq U$. Then $V=\left[f(U^c)\right]^c$ is neutrosophic grb−open set containing W such that $f^{-1}(V)\subseteq U$. Conversely, let W be a neutrosophic closed set of X. Then $f^{-1}\left[\left(f(W)\right)^c\right]\subseteq W^c$ and W^c is neutrosophic open in X. By assumption, there exists a neutrosophic grb−open set V of Y such that $\left[f(W)\right]^c\subseteq V$ and $f^{-1}(V)\subseteq W^c$ and so $W\subseteq\left[f^{-1}(V)\right]^c$. Hence $V^c\subseteq f(W)\subseteq f\left[\left(f^{-1}(V)\right)^c\right]\subseteq V^c$, which implies $f(W)=V^c$. Since V^c is neutrosophic grb−closed, $f(W)$ is neutrosophic grb−closed and f is neutrosophic grb−closed mapping.

Theorem 5.5. Let $f:(X,T_N)\to(Y,\sigma_N)$ be a neutrosophic closed mapping and $g:(Y,\sigma_N)\to(Z,\eta_N)$ be a neutrosophic grb−closed mapping. Then their composition $gof:(X,T_N)\to(Z,\eta_N)$ is neutrosophic grb−closed.

Proof. Let F be a neutrosophic closed set in X. Since f is neutrosophic closed, $f(F)$ is neutrosophic closed in Y. Since g is neutrosophic grb−closed, $g[f(F)]=(gof)(F)$ is neutrosophic grb−closed in Z. Hence gof is a neutrosophic grb−closed mapping.

Theorem 5.6. Let $f:(X,T_N)\to(Y,\sigma_N)$ and $g:(Y,\sigma_N)\to(Z,\eta_N)$ be two mappings such that their composition $gof:(X,T_N)\to(Z,\eta_N)$ is neutrosophic grb−closed. Then the following statements are true.

(i) If f is neutrosophic continuous and surjective, then g is neutrosophic grb−closed.

(ii) If g is neutrosophic grb−irresolute and injective, then f is neutrosophic grb−closed.

Proof. (i) Let A be a neutrosophic closed set of Y. Since $f^{-1}(A)$ neutrosophic continuousis f is neutrosophic closed in X. Since gof is neutrosophic grb−closed, $(gof)\left(f^{-1}(A)\right)$ is neutrosophic grb−closed in Z. Since f is surjective, $g(A)$ is neutrosophic grb−closed in Z. Hence g is neutrosophic grb−closed.

(ii) Let B be any neutrosophic closed set of X. Since gof is neutrosophic grb−closed, $(gof)(B)$ is neutrosophic grb−closed in Z. Since g is neutrosophic grb−irresolute, $g^{-1}(gof(B))$ is neutrosophic grb−closed in Y. Since g is injective, $f(B)$ is neutrosophic grb−closed in Y. Hence f is neutrosophic grb−closed.

Theorem 5.7. Let $f:(X,T_N)\to(Y,\sigma_N)$ be a neutrosophic grb−closed mapping.

(i) If A is neutrosophic closed set of X, then the restriction $f_A:A\to Y$ is neutrosophic grb−closed.

(ii) If $A=f^{-1}(B)$ for some neutrosophic closed set B of Y, then the restriction $f_A:A\to Y$ is neutrosophic grb−closed.

Proof. (i) Let B be any neutrosophic closed set of A. Then $B = A \cap F$ for some neutrosophic closed set F of X and so B is neutrosophic closed in X. By hypothesis, $f(B)$ is neutrosophic $grb-$closed in Y. But $f(B) = f_A(B)$, therefore f_A is a neutrosophic $grb-$closed mapping.

(ii) Let D be any neutrosophic closed set of A. Then $D = A \cap H$ for some neutrosophic closed set H in X. Now,

$$f_A(D) = f(D) = f(A \cap H) = f\left[f^{-1}(B) \cap H\right]$$
$$= B \cap f(H).$$

Since f is a neutrosophic $grb-$closed mapping, so $f(H)$ is a neutrosophic $grb-$closed set in Y. Hence f_A is a neutrosophic $grb-$closed mapping.

Theorem 5.8. A function $f:(X, T_N) \to (Y, \sigma_N)$ is neutrosophic $grb-$open if and only if $f\left[N_{eu}Int(A)\right] \subseteq N_{eu}grbInt\left[f(A)\right]$, for every neutrosophic set A of X.

Proof. Suppose $f:(X, T_N) \to (Y, \sigma_N)$ is a neutrosophic $grb-$open function and A is any neutrosophic set in X. Then $N_{eu}Int(A)$ is a neutrosophic open set in X. Since f neutrosophic $grb-$open, $f\left[N_{eu}Int(A)\right]$ is a neutrosophic $grb-$open set. Since $N_{eu}grbInt\left[f\left(N_{eu}IntA\right)\right] \subseteq N_{eu}grbInt\left[f(A)\right]$, $f\left[N_{eu}Int(A)\right] \subseteq N_{eu}grbInt\left[f(A)\right]$.

Conversely, $f\left[N_{eu}Int(A)\right] \subseteq N_{eu}grbInt\left[f(A)\right]$ for every neutrosophic set A in X. Let U be a neutrosophic open set in X. Then $N_{eu}Int(U) = U$ and by hypothesis, $f(U) \subseteq N_{eu}grbInt\left[f(U)\right]$. But $N_{eu}grbInt\left[f(U)\right] \subseteq f(U)$. Therefore, $f(U) = N_{eu}grbInt\left[f(U)\right]$. Then $f(U)$ is neutrosophic $grb-$open. Hence f is a neutrosophic $grb-$open mapping.

Theorem 5.9. A function $f:(X, T_N) \to (Y, \sigma_N)$ is neutrosophic $grb-$open if and only if for each $x_{(r,s,t)} \in X$ and for each neutrosophic neighborhood U of $x_{(r,s,t)}$ in X, there exists a neutrosophic $grb-$neighborhood W of $f\left(x_{(r,s,t)}\right)$ in Y such that $W \subseteq f(U)$.

Proof. Let $f:(X, T_N) \to (Y, \sigma_N)$ be a neutrosophic $grb-$open function. Let $x_{(r,s,t)} \in X$ and U be any arbitrary neutrosophic neighborhood of $x_{(r,s,t)}$ in X. Then there exists a neutrosophic open set G such that $x_{(r,s,t)} \in G \subseteq U$. By Theorem 5.8, $f(G) = f\left[N_{eu}Int(G)\right] \subseteq N_{eu}grbInt\left[f(G)\right]$. But, $N_{eu}grbInt\left[f(G)\right] \subseteq f(G)$. Therefore, $N_{eu}grbInt\left[f(G)\right] = f(G)$ and hence $f(G)$ is neutrosophic $grb-$open in Y. Since $x_{(r,s,t)} \in G \subseteq U$, $f\left(x_{(r,s,t)}\right) \in f(G) \subseteq f(U)$ and so the result follows by taking $W = f(G)$.

Conversely, Let U be any neutrosophic open set in X. Let $x_{(r,s,t)} \in U$ and $f\left(x_{(r,s,t)}\right) = y_{(k,l,m)}$. Then by assumption there exists a neutrosophic $grb-$neighborhood $W_{\left(y_{(k,l,m)}\right)}$ of $y_{(k,l,m)}$ in Y such that $W_{\left(y_{(k,l,m)}\right)} \subseteq f(U)$. Since $W_{\left(y_{(k,l,m)}\right)}$ is a neutrosophic $grb-$neighborhood of $y_{(k,l,m)}$, there exists a neutrosophic $grb-$open set $V_{\left(y_{(k,l,m)}\right)}$ in Y such that $y_{(k,l,m)} \in V_{\left(y_{(k,l,m)}\right)} \subseteq W_{\left(y_{(k,l,m)}\right)}$. Therefore, $f(U) = \bigcup\left\{V_{\left(y_{(k,l,m)}\right)} : y_{(k,l,m)} \in f(U)\right\}$. Since the union of neutrosophic $grb-$open sets is neutrosophic $grb-$open, $f(U)$ is a neutrosophic $grb-$open set in Y. Thus, f is a neutrosophic $grb-$open mapping.

Theorem 5.10. For any bijective mapping $f:(X, T_N) \to (Y, \sigma_N)$ the following statements are equivalent:

(i) $f^{-1}: Y \to X$ is neutrosophic $grb-$continuous.

(ii) f is neutrosophic $grb-$open.

(iii) f is neutrosophic $grb-$closed.

Proof. (i)$\Rightarrow$(ii): Let U be a neutrosophic open set in X. By assumption, $\left(f^{-1}\right)^{-1}(U)=f(U)$ is neutrosophic grb–open in Y and so f is neutrosophic grb–open.

(ii)$\Rightarrow$(iii): Let F be a neutrosophic closed set of X. Then F^c is a neutrosophic open set in X. By assumption $f\left(F^c\right)$ is neutrosophic grb–open in Y. But $f\left(F^c\right)=\left[f(F)\right]^c$. Therefore $f(F)$ is neutrosophic grb–closed set in Y. Hence, f is neutrosophic grb–closed.

(iii)$\Rightarrow$(i): Let F be a neutrosophic closed set of X. By assumption, $f(F)$ is neutrosophic grb–closed set in Y. But $f(F)=\left(f^{-1}\right)^{-1}(F)$ and therefore by Theorem 3.4, $f^{-1}:Y\to X$ is neutrosophic grb–continuous.

VI. STRONGLY NEUTROSOPHIC grb-CONTINUOUS AND PERFECTLY grb-CONTINUOUS MAPPINGS

In this section, we introduce and study the concepts of strongly neutrosophic grb–continuous and perfectly neutrosophic grb–continuous mappings in neutrosophic topological spaces.

Definition 6.1. A mapping $f:(X,\mathrm{T}_N)\to(Y,\sigma_N)$ is called strongly neutrosophic grb–continuous if the inverse image of every neutrosophic grb–open set in Y is neutrosophic open in X.

Definition 6.2. A mapping $f:(X,\mathrm{T}_N)\to(Y,\sigma_N)$ is called perfectly neutrosophic grb–continuous if the inverse image of every neutrosophic grb–open set in Y is neutrosophic clopen in X.

Theorem 6.3. Let $f:(X,\mathrm{T}_N)\to(Y,\sigma_N)$ be a mapping. Then the following statements are true:

(i) If f is perfectly neutrosophic grb–continuous, then f is perfectly neutrosophic continuous.

(ii) If f is strongly neutrosophic grb–continuous, then f is neutrosophic continuous.

Proof. (i) Let $f:X\to Y$ be perfectly neutrosophic grb–continuous. Let V be a a neutrosophic open set in Y. Then V is neutrosophic grb–open set in Y. Since f is perfectly neutrosophic grb–continuous, $f^{-1}(V)$ is neutrosophic clopen in X. Hence f is perfectly neutrosophic continuous.

(ii) Let $f:X\to Y$ be strongly neutrosophic grb–continuous. Let G be aneutrosophic open set in Y. Then G is neutrosophic grb–open set in Y. Since f is strongly neutrosophic grb–continuous, $f^{-1}(G)$ is neutrosophic open in X. Therefore f is neutrosophic continuous.

Theorem 6.4. Let $f:X\to Y$ be strongly neutrosophic grb–continuous and A be a neutrosophic open set in Y. Then the restriction map, $f_A:A\to Y$ is strongly neutrosophic grb–continuous.

Proof. Let V be any neutrosophic grb–open set in Y. Since f is strongly neutrosophic grb–continuous, $f^{-1}(V)$ is neutrosophic open in X. But $f_A^{-1}(V)=A\cap f^{-1}(V)$. Since A and $f^{-1}(V)$ are neutrosophic open, $f_A^{-1}(V)$ is neutrosophic open in A. Hence f_A is strongly neutrosophic grb–continuous.

Theorem 6.5. Every perfectly neutrosophic grb–continuous mapping $f:(X,\mathrm{T}_N)\to(Y,\sigma_N)$ is strongly neutrosophic grb–continuous.

Proof. Let $f:X\to Y$ be perfectly neutrosophic grb–continuous and V be neutrosophic grb–open set in Y. Since f is perfectly neutrosophic grb–continuous, $f^{-1}(V)$ is neutrosophic clopen in X. That is both neutrosophic open and neutrosophic closed in X. Hence f is strongly neutrosophic grb–continuous.

Theorem 6.6. If $f:(X,\mathrm{T}_N)\to(Y,\sigma_N)$ and $g:(Y,\sigma_N)\to(Z,\eta_N)$ are strongly neutrosophic grb–continuous, then $gof:(X,\mathrm{T}_N)\to(Z,\eta_N)$ is also strongly neutrosophic grb–continuous.

Proof. Let V be a neutrosophic open set in Z. Since g is a strongly neutrosophic $grb-$continuous mapping, $g^{-1}(V)$ is neutrosophic open in Y. Then $g^{-1}(V)$ is neutrosophic open in Y. Since f is a strongly neutrosophic $grb-$continuous mapping, $f^{-1}\left[g^{-1}(V)\right]=(gof)^{-1}(V)$ is neutrosophic open in X. Therefore, gof is strongly neutrosophic $grb-$continuous.

Theorem 6.7. If $f:(X,T_N)\to(Y,\sigma_N)$ and $g:(Y,\sigma_N)\to(Z,\eta_N)$ are perfectly neutrosophic $grb-$continuous mappings, then their composition $gof:(X,T_N)\to(Z,\eta_N)$ is also perfectly neutrosophic $grb-$continuous mapping.

Proof. Let V be a neutrosophic $grb-$open set in Z. Since g is a perfectly neutrosophic $grb-$continuous mapping, $g^{-1}(V)$ is neutrosophic clopen in Y. That is $g^{-1}(V)$ both neutrosophic open and neutrosophic closed in X. Then $g^{-1}(V)$ is neutrosophic $grb-$open set in X. Since f is a perfectly neutrosophic $grb-$continuous mapping, $f^{-1}\left[g^{-1}(V)\right]=(gof)^{-1}(V)$ is neutrosophic clopen in X. Therefore gof is perfectly neutrosophic $grb-$continuous.

Theorem 6.8. Let $f:(X,T_N)\to(Y,\sigma_N)$ and $g:(Y,\sigma_N)\to(Z,\eta_N)$ be mappings. Then the following statements are true.

(i) If g is strongly neutrosophic $grb-$continuous and f is neutrosophic $grb-$continuous, then gof is neutrosophic $grb-$irresolute.

(ii) If g is perfectly neutrosophic $grb-$continuous and f is neutrosophic continuous, then gof is strongly neutrosophic $grb-$continuous.

(iii) If g is strongly neutrosophic $grb-$continuous and f is perfectly neutrosophic $grb-$continuous, then gof is perfectly neutrosophic $grb-$continuous.

(iv) If g is neutrosophic $grb-$continuous and f is strongly neutrosophic $grb-$continuous, then gof is neutrosophic continuous.

Proof. (i) Let V be a neutrosophic $grb-$open set in Z. Since g is a strongly neutrosophic $grb-$continuous mapping, $g^{-1}(V)$ is neutrosophic open set in Y. Since f is a neutrosophic $grb-$continuous mapping, $f^{-1}\left[g^{-1}(V)\right]=(gof)^{-1}(V)$ is neutrosophic $grb-$open set in X. Hence gof is neutrosophic $grb-$irresolute. (ii) Let V be a neutrosophic $grb-$open set in Z. Since g is a perfectly neutrosophic $grb-$continuous mapping, $g^{-1}(V)$ is neutrosophic clopen set in Y. That is, $g^{-1}(V)$ is both neutrosophic open and neutrosophic closed. Since f is a neutrosophic $grb-$continuous mapping, $f^{-1}\left[g^{-1}(V)\right]=(gof)^{-1}(V)$ is neutrosophic open in X. Therefore gof is strongly neutrosophic $grb-$continuous.

(iii) Let V be a neutrosophic $grb-$open set in Z. Since g is a strongly neutrosophic $grb-$continuous mapping, $g^{-1}(V)$ is neutrosophic open set in Y. Since every neutrosophic open set is neutrosophic $grb-$open set. So $g^{-1}(V)$ is neutrosophic $grb-$open set in X. Since f is a perfectly neutrosophic $grb-$continuous mapping, $f^{-1}\left[g^{-1}(V)\right]=(gof)^{-1}(V)$ is neutrosophic clopen in X. Hence gof is perfectly neutrosophic $grb-$continuous.

(iv) Let V be a neutrosophic open set in Z. Since g is a neutrosophic $grb-$continuous mapping, $g^{-1}(V)$ is neutrosophic $grb-$open set in Y. Since f is a strongly neutrosophic $grb-$continuous map, $f^{-1}\left[g^{-1}(V)\right]=(gof)^{-1}(V)$ is neutrosophic open in X. So gof is neutrosophic continuous.

VII. NEUTROSOPHIC CONTRA grb-CONTINUOUS MAPPINGS AND NEUTROSOPHIC CONTRA grb-IRRESOLUTE MAPPINGS

In this section, we introduce the concepts of neutrosophic contra $grb-$continuous mappings and neutrosophic contra $grb-$irresolute mappings and investigate their fundamental properties and characterizations.

Definition 7.1. A mapping $f:(X,T_N)\rightarrow(Y,\sigma_N)$ is said to be neutrosophic contra $-$continuous if the inverse image of every neutrosophic open set in Y is neutrosophic closed set in X.

Definition 7.2. A mapping $f:(X,T_N)\rightarrow(Y,\sigma_N)$ is called neutrosophic contra $grb-$continuous if the inverse image of every neutrosophic open set in Y is neutrosophic $grb-$closed in X.

Theorem 7.3. Let $f:(X,T_N)\rightarrow(Y,\sigma_N)$ be a neutrosophic contra $-$continuous mapping. Then f is neutrosophic contra $grb-$continuous.

Proof. Let V be any neutrosophic open set in Y. Since f is neutrosophic contra continuous, $f^{-1}(V)$ is neutrosophic closed set in X. As every neutrosophic closed set is neutrosophic $grb-$closed, we have $f^{-1}(V)$ is neutrosophic $grb-$closed set in X. Therefore f is neutrosophic contra $grb-$continuous.

Theorem 7.4. A mapping $f:(X,T_N)\rightarrow(Y,\sigma_N)$ is neutrosophic contra $grb-$continuous if and only if the inverse image of every neutrosophic closed set in Y is neutrosophic $grb-$open set in X.

Proof. Let V ba a neutrosophic closed set in Y. Then V^C is neutrosophic open set in Y. Since f is neutrosophic contra $grb-$continuous, $f^{-1}(V^C)$ is neutrosophic $grb-$closed set in X. But $f^{-1}(V^C)=1-f^{-1}(V)$ and so $f^{-1}(V)$ is neutrosophic $grb-$open set in X. Conversely, assume that the inverse image of every neutrosophic closed set in Y is neutrosophic $grb-$open in X. Let W be a neutrosophic open set in Y. Then W^C is neutrosophic closed in Y. By hypothesis $f^{-1}(W^C)=1-f^{-1}(W)$ is neutrosophic $grb-$open in X, and so $f^{-1}(W)$ is neutrosophic $grb-$closed set in X. Thus f is neutrosophic contra

Theorem 7.5. If a mapping $f:(X,T_N)\rightarrow(Y,\sigma_N)$ is neutrosophic contra $grb-$continuous and $g:(Y,\sigma_N)\rightarrow(Z,\eta_N)$ is neutrosophic continuous, then their composition $gof:(X,T_N)\rightarrow(Z,\eta_N)$ is neutrosophic contra $grb-$continuous.

Proof. Let W ba a neutrosophic open set in Z. Since g is neutrosophic continuous, $g^{-1}(W)$ is neutrosophic open set in Y. Since f is neutrosophic contra $grb-$continuous, $f^{-1}\left[g^{-1}(W)\right]$ is neutrosophic $grb-$closed set in X. But $(gof)^{-1}(W)=$ $f^{-1}\left[g^{-1}(W)\right]$. Thus gof is neutrosophic contra $grb-$continuous.

Definition 7.6. A mapping $f:(X,T_N)\rightarrow(Y,\sigma_N)$ is called neutrosophic contra $grb-$irresolute if the inverse image of every neutrosophic $grb-$open set in Y is neutrosophic $grb-$closed in X.

Theorem 7.7. If a mapping $f:(X,T_N)\rightarrow(Y,\sigma_N)$ is neutrosophic contra $grb-$irresolute, then it is neutrosophic contra $grb-$continuous.

Proof. Let V be a neutrosophic open set in Y. Since every neutrosophic open set is neutrosophic $grb-$open, V is neutrosophic $grb-$open set in Y. Since f is neutrosophic contra $grb-$irresolute, $f^{-1}(V)$ is neutrosophic $grb-$closed set in X. Thus f is neutrosophic contra $grb-$continuous.

Theorem 7.8. Let (X,T_N), (Y,σ_N) and (Z,η_N) be neutrosophic topological spaces. If $f:(X,T_N)\rightarrow(Y,\sigma_N)$ is neutrosophic contra $grb-$irresolute and $g:(Y,\sigma_N)\rightarrow(Z,\eta_N)$ is neutrosophic $grb-$continuous, then $gof:(X,T_N)\rightarrow(Z,\eta_N)$ is neutrosophic contra $grb-$continuous.

Proof. Let W be any neutrosophic open set in Z. Since g is neutrosophic grb–continuous, $g^{-1}(W)$ is neutrosophic grb–open set in Y. Since f is neutrosophic contra grb–irresolute, $f^{-1}\left[g^{-1}(W)\right]$ is neutrosophic grb–closed set in X. But $(gof)^{-1}(W)=f^{-1}\left[g^{-1}(W)\right]$. Thus gof is neutrosophic contra grb–continuous.

Theorem 7.9. If $f:(X,T_N)\rightarrow(Y,\sigma_N)$ is neutrosophic grb–irresolute and $g:(Y,\sigma_N)\rightarrow(Z,\eta_N)$ is neutrosophic contra grb–irresolute, then their composition $gof:(X,T_N)\rightarrow(Z,\eta_N)$ is neutrosophic contra grb–irresolute mapping.

Proof. Let W be any neutrosophic grb–open set in Z. Since g is neutrosophic contra grb–irresolute, $g^{-1}(W)$ is neutrosophic grb–closed set in Y. Since f is neutrosophic grb–irresolute, $f^{-1}\left[g^{-1}(W)\right]$ is neutrosophic grb–closed set in X. But $(gof)^{-1}(W)=f^{-1}\left[g^{-1}(W)\right]$. Thus gof is neutrosophic contra grb–irresolute.

VIII. CONCLUSION

In this research article, we have introduced and studied the properties of neutrosophic grb–continuous functions, neutrosophic grb–irresolute functions, neutrosophic grb–closed functions, neutrosophic grb–open functions, strongly neutrosophic grb–continuous functions, perfectly neutrosophic grb–continuous functions, neutrosophic contra grb–continuous functions, and neutrosophic contra grb–irresolute functions in neutrosophic topological spaces and established the relations between them. We have obtained fundamental characterizations of theses mappings and investigated preservation properties. We expect the results in this chapter will be basis for further applications of mappings in neutrosophic topological spaces.

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