Neutrosophic grb-Continuous and grb-Irresolute Mappings in Neutrosophic Topological Spaces

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*Abstract***— Real-life structures always include indeterminacy. The Mathematical tool which is well known in dealing with indeterminacy is neutrosophic. Smarandache proposed the approach of neutrosophic sets. Neutrosophic sets deal with uncertain data. The notion of neutrosophic set is generally referred to as the generalization of intuitionistic fuzzy set. In 2021, Dr. G. Sindhu introduced the concept of Neutrosophic generalized regular b-closed sets and neutrosophic generalized b-open sets and presented some of their properties in Neutrosophic topological spaces. In this research paper, we introduce the concepts of neutrosophic** grb**-**continuous **mappings, neutrosophic** grb**-**irresolute **mappings, neutrosophic** grb**-**closed **mappings, neutrosophic** grb**-**open **mappings,**

strongly neutrosophic grb**-**continuous **mappings, perfectly neutrosophic** grb**-**continuous **mappings, neutrosophic contra** grb**-**continuous **mappings and neutrosophic contra** grb**-**irresolute **mappings in neutrosophic topological spaces. We investigate and obtain several properties and characterizations concerning these mappings in neutrosophic topological spaces.**

*Keywords***: Neutrosophic topological space; Neutrosophic** grb**-**open **set; Neutrosophic** grb**-**closed **set; Neutrosophic** grb**-**continuous **mapping; Neutrosophic** grb**-**irresolute **mapping; Neutrosophic** grb**-**open **mapping; Neutrosophic** grb**-**closed **mapping; Strongly neutrosophic** grb**-**continuous **mapping; Perfectly neutrosophic** grb**-**continuous **mapping; Neutrosophic contra** grb**-**continuous **mapping; Neutrosophic contra** grb**-**irresolute **mapping**

I.INTRODUCTION

Many real-life problems in Business, Finance, Medical Sciences, Engineering, and Social Sciences deal with uncertainties. Smarandache studies neutrosophic set as an approach for solving issues that cover unreliable, indeterminacy, and persistent data. Applications of neutrosophic topology depend upon the properties of neutrosophic closed sets, neutrosophic open sets, neutrosophic interior operator, neutrosophic closure operator, and neutrosophic sets. In 2021, Dr. G. Sindhu introduced the concepts of Neutrosophic generalized regular bclosed sets and Neutrosophic generalized b-open sets and presented some some of their properties in Neutrosophic topological spaces. We introduce the concepts of neutrosophic grb**-**continuous mappings, neutrosophic grb**-**irresolute mappings, neutrosophic grb**-**closed neutrosophic grb**-**open strongly neutrosophic grb**-**continuous mappings, perfectly neutrosophic *grb***-**continuous neutrosophic contra grb**-**continuous mappings and neutrosophic contra grb**-**irresolute mappings in neutrosophic topological spaces. We investigate and obtain several properties and characterizations concerning these mappings in neutrosophic topological spaces.

II. PRELIMINARIES

Definition 2.1. Let *X* be a non-empty fixed set. A neutrosophic set P is an object having the form $P = \{ \langle x, \mu_P(x), \sigma_P(x), \gamma_P(x) \rangle : x \in X \},\$

where $\mu_{P}(x)$ -represents the degree of membership, $\sigma_p(x)$ -represents the degree of indeterminacy, and $\gamma_{P}(x)$ -represents the degree of non-membership.

Definition 2.2. A neutrosophic topology on a nonempty set *X* is a family T_N of neutrosophic subsets

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of *X* satisfying (i) 0_N , $1_N \in T_N$. (ii) $G \cap H \in T_N$ for every $G, H \in T_N$, $(iii) \bigcup_{j \in J} G_j \in T_N$ *G* \in $\in T_{N}$ for $\text{every}\big\{G_j : j \in J\big\} \subseteq \tau_N.$

Then the pair (X,T_N) is called a neutrosophic topological space. The elements of T_N are called neutrosophic open sets in *X*. A neutrosophic set *A* is called a neutrosophic closed set if and only if its complement A^c is a neutrosophic open set.

Definition 2.3. Let (X,T_N) be a neutrosophic topological space and *A* be a neutrosophic set. Then

 i The neutrosophic interior of *A*, denoted by $N_{eu} Int(A)$ is the union of all neutrosophic open subsets of *X* contained in *A*.

(ii) The neutrosophic closure of A denoted by $N_{eu}Cl(A)$ is the intersection of all neutrosophic closed sets containing *A*.

Definition 2.4. Let A be a neutrosophic set in a neutrosophic topological space (X, T_N) . Then the set *A* is called a neutrosophic regular open set in a neutrosophic topological space *X* if reutrosophic topologic
 $A \subseteq N_{eu} Int \Big[N_{eu} Cl(A) \Big].$

Definition 2.5. Let A be a neutrosophic set in a neutrosophic topological space (X, T_N) . Then the set *A* is called a neutrosophic α -open set in neutrosophic topological space *X* if utrosophic topological spa
 $\subseteq N_{eu} Int\left[N_{eu} Cl(N_{eu} In(A))\right]$. reutrosophic topological spies
 $A \subseteq N_{eu} Int \Big[N_{eu} Cl(N_{eu} In(A)) \Big].$
 Definition 2.6. Let A be a neutrose

Let *A* be a neutrosophic set in a neutrosophic topological space (X, T_N) . Then the set A is called a neutrosophic b-open set in neutrosophic topological space (X, T_N) if neutrosophic topological space (X, T_N) if
 $A \subseteq N_{eu} Int\big[\big[\big[\big[\big(\mathcal{A}\big]\big]\big] \cup \big[\big] \big] \cup \big[\big[\big[\big(\mathcal{A}\big]\big]\big]$. *eutrosophic* topological space (X, I_N) *if*
 $A \subseteq N_{eu} Int[N_{eu} Cl(A)] \cup N_{eu} Cl[N_{eu} Int(A)].$
 Definition 2.7. Let A be a neutrosophic set in a

Let *A* be a neutrosophic set in a neutrosophic topological space (X, T_N) . Then the set *A* is called a Neutrosophic Generalized Regular **b**-closed (briefly grb-closed) set in neutrosophic topological space (X, T_N) if $N_{eu}bCl(A) \subseteq U$

whenever $A \subseteq U$ and U is neutrosophic regular open set in *X*.

Definition 2.8. Let A be a neutrosophic set in a neutrosophic topological (X, T_N) . Then the set A is called a Neutrosophic Generalized Regular b-open (briefly grb -open) set in neutrosophic topological

 (X, T_N) if the complement A^C of A is neutrosophic *grb***-**closed set in *X*.

Definition 2.9. Let A be a subset of a neutrosophic topological (X, T_N) . Then neutrosophic generalized
regular **b**-interior of *A* is given by:
 N_{eu} grbInt (A) = regular **b**-interior of *A* is given by: $\{G : G \text{ is a } \mathbb{N}_{eu} \text{ such that } G \subseteq A\}.$ regular b —inte
 N_{eu} *grbInt* (A) =

Definition 2.10. Let *A* be a subset of a neutrosophic topological $(X, T_{N}).$ Then neutrosophic generalized regular b-closure of A is N_{eu} grb $Cl(A)$ = c *ropolog*
 c *rbCl*(*A*) =

 $G : G$ is a neutrosophic grb-closed set in X $\begin{bmatrix} G : G \end{bmatrix}$ *and* $A \subseteq G$
 $\begin{cases} G : G \text{ is a neutrosophic grb -closed set in X} \\ and A \subseteq G \end{cases}$ g **-**

Remark 2.11. Let A be a subset of a neutrosophic topological (X, T_N) . Then N_{eu} *grbInt* (A) is neutrosophic grb – open set in (X, T_N) . The complement of N_{eu} *grbInt* (A) is N_{eu} *grb*Cl (A) .

Theorem 2.12. Every neutrosophic closed (resp. open) set in a neutrosophic topological space is neutrosophic *grb***-**closed

resurosophic *grb*-open *)* set.

Theorem 2.13. Every neutrosophic α -closed set in a neutrosophic topological space (X, T_N) is neutrosophic *grb***-**closed set.

Theorem 2.14. The union of any two neutrosophic *grb***-**closed sets in a neutrosophic topological space (X, T_N) is also a neutrosophic grb-closed set in (X, T_N) .

Theorem 2.15. The intersection of any two neutrosophic *grb***-**open sets in a neutrosophic topological space (X, T_N) is also a neutrosophic grb-open set in (X, T_N) .

Theorem 2.16. The union of any family of neutrosophic *grb***-**open sets in a neutrosophic topological space (X, T_N) is also a neutrosophic *grb***-**open set in (X, T_N) .

Definition 2.17. Let A be a neutrosophic subset of a neutrosophic topological space (X, T_N) . Then the neutrosophic *grb***-**frontier of a neutrosophic subset *A* of X is denoted by N_{eu} grb $Fr(A)$ and is defined by grb-frontier of a neutrosophic subset A of $\begin{array}{c} J(V) \ J' \to V \end{array}$
 $\begin{array}{c} N \to N_{eu} \text{grb} \text{Fr}(A) \end{array}$ and is defined $\begin{array}{c} N \to N_{eu} \text{grb} \text{Cl}(A) \cap N_{eu} \text{grb} \text{Cl}(A^c). \end{array}$ is denoted by $N_{eu} \text{grb} \text{Fr}(A)$ and is def
 $N_{eu} \text{grb} \text{Fr}(A) = N_{eu} \text{grb} \text{Cl}(A) \cap N_{eu} \text{grb} \text{Cl}(A^C)$

Theorem 2.18. For a neutrosophic set A in a neutrosophic

topological space *X*,*^N* , the following statements are true: ⁱ . N N *grbInt A grbCl A C C eu eu* ii . *C C eu Cl A eu* N N *grb grbInt A*

Definition 2.19. Let $f:(X, T_N) \rightarrow (Y, \sigma_N)$ be a mapping. Then f is called a neutrosophic continuous mapping if $f^{-1}(V)$ is a neutrosophic open set in *X* for every neutrosophic open set *V* in *Y*.

Theorem 2.20. Let $f:(X, T_X) \rightarrow (Y, \sigma_X)$ be a mapping. Then f is called a neutrosophic continuous mapping if $f^{-1}(V)$ is a neutrosophic closed set in *X* for every neutrosophic closed set *V* in *Y*.

III. NEUTROSOPHIC grb-CONTINUOUS MAPPINGS

In this section, we introduce the concepts of neutrosophic *grb***-**continuous mappings in neutrosophic topological spaces. Also, we study some of the main results depending on neutrosophic *grb***-**open sets.

Definition 3.1. Let $f:(X,T_N) \to (Y,\sigma_N)$ be a mapping. Then f is called a neutrosophic *grb***-**continuous mapping if $f^{-1}\big(V\big)$. is a neutrosophic *grb***-**open set in in X for every neutrosophic open set *V* in *Y*.

Theorem 3.2. Every neutrosophic continuous mapping is neutrosophic *grb***-**continuous mapping.

Proof. Let $f: (X, T_N) \to (Y, \sigma_N)$ be neutrosophic continuous mapping. Let V be a neutrosophic open set in $(Y, \sigma_{\scriptscriptstyle N}^{})$. Then $f^{-1}(V)$ is neutrosophic open set in (X, T_N) . Since every neutrosophic open set is neutrosophic *grb***-**open, $f^{-1}(V)$ is neutrosophic grb-open set in (X, T_N) . Hence f is neutrosophic gzb-continuous mapping. **Theorem 3.3.** Let (X, T_N) , (Y, σ_N) and (Z, η_N) be neutrosophic topological spaces. If $f: (X, T_{N}) \rightarrow (Y, \sigma_{N})$ is a neutrosophic *grb***-**continuous mapping and $g:(Y,\sigma_{N})\rightarrow (Z,\eta_{N})$ is neutrosophic grb-continuous, then $gof: (X,T_N) \to (Z,\eta_N)$ is a is neutrosophic *grb***-**continuous mapping.

Proof. Let G be a neutrosophic open set in *Z*. Since $g:(Y,\sigma_{N})\rightarrow (Z,\eta_{N})$ is neutrosophic continuous, $f^{-1}(G)$ is neutrosophic open in Y. Since *f* is a neutrosophic *grb***-**continuous mapping, $f^{-1}\left[f^{-1}(G)\right]$ is neutrosophic *grb***-**open in *X*. But gn:b-open in
 $f^{-1}[g^{-1}(G)] = (gof)^{-1}(G)$. Then $(gof)^{-1}(G)$ is neutrosophic *grb*-open set in *X*. Hence, *gof* is a neutrosophic *grb***-**continuous mapping.

Theorem 3.4. Let (X, T_N) and (Y, σ_N) be two neutrosophic topological spaces. Then prove that $f: (X, T_{N}) \rightarrow (Y, \sigma_{N})$ is neutrosophic grb-continuous if and only if $f^{-1}(B)$ $f^{-1}(B)$ is neutrosophic *grb***-**closed set in *X* for every neutrosophic closed set *B* in *Y*.

Proof. Let *B* be a neutrosophic closed set in *Y*. Then B^C is neutrosophic open set in Y. Since f is neutrosophic *grb*-continuous. Therefore $f^{-1}(B^C)$ is a neutrosophic *grb***-**open set in *X*. Since $f^{-1}(B^C) = [f^{-1}(B)]^C$, $f^{-1}(B)$ **i** neutrosophic *grb***-**closed set in *X*.

Conversely, Let *B* be a neutrosophic open set in *Y*. Then B^C is neutrosophic closed set in *Y*. By assumption $f^{-1}(B^c)$ is neutrosophic grb-closed set in *X*. Since $f^{-1}(B^c) = [f^{-1}(B)]^c$, $f^{-1}(B)$ is neutrosophic *grb***-**open set in *X*. Hence *f* is neutrosophic *grb***-**continuous.

Theorem 3.5. Let (X, T_N) and (Y, σ_N) be two neutrosophic topological spaces and $f: X \to Y$ be a mapping. Then *f* a neutrosophic *grb***-**continuous mapping if and only if *f* (N_{eu} *grb* \leq *CON*(*A*)) $\leq N_{eu}$ *Grb Cl*(*f*(*A*)) for example if and only for every neutrosophic set *A* in *X*.

Proof. Let A be a neutrosophic set in X and *f* be a neutrosophic *grb***-**continuous mapping. *f* be a neutrosophic *grb*-continuous mapping.
Then evidently $f(A) \subseteq N$ _{eu} grbcl[$f(A)$]. Now, Then evidently $f(A) \subseteq N$ equals the *A* f and $A \subseteq f^{-1}[f(A)] \subseteq f^{-1}[\text{N}$ equals $A \subseteq f^{-1}[f(A)] \subseteq f^{-1}[\text{N}$ equals $A \subseteq f^{-1}[f(A)]$ and $_{eu}$ *grbCl* $(f(A))$ and N_{eu} *grbCl* $(A) \subseteq$ \equiv
 $\left(\begin{smallmatrix} N & \text{curl } &$

 $\left[\left(\mathbb{N}_{\text{eu}} \text{grkCl}(f(A)) \right) \right].$ N_{eu} grb $Cl[f^{-1}(N_{eu}$ grb $Cl(f(A))]$. Since f is a neutrosophic N _{eu} grb-continuous mapping and N_{eu} *grb* $Cl[f(A)]$ is a neutrosophic *grb*-closed set. Thus $\text{dist}(I[f(A)] \text{ is a neutrosophic } \text{grb}-\text{closed set.}$
 $\text{us} \qquad \text{N}_{eu} \text{grtCl}\Big[f^{-1}(\text{N}_{eu} \text{grtCl}(f(A))\Big)\Big] = \text{in}$
 $\text{Hence,} \qquad \text{if} \qquad \text{Simplies}$ f^{-1} $\left[N_{eu}\text{grbCl}(f(A))\right]$. Hence, f^{-1} $\left[\begin{array}{l}\mathbb{N} \mathbb{R} \mathbb{Z} \mathbb{Z} \mathbb{Z} \mathbb{Z} \mathbb{Z} \left(f(A) \right) \right].$
 $f\left[\begin{array}{l}\mathbb{N} \mathbb{Z} \mathbb{Z} \mathbb{Z} \mathbb{Z} \end{array} \left(A \right) \right] \subseteq \mathbb{N} \mathbb{Z} \mathbb{Z} \mathbb{Z} \mathbb{Z} \mathbb{Z} \left[f\left(A \right) \right].$

Conversely, let $\begin{aligned} &\text{Conversely,} \\ &f\left[\mathbb{N}_{eu}\text{grbCl}(A)\right] \subseteq \mathbb{N}_{eu}\text{grbCl}\left[f\left(A\right)\right], \text{ for } \end{aligned}$ for each neutrosophic set A in X . Let F be a neutrosophic closed set in Then A in X. Let F be a neutrosophic set in Y .
 $(f^{-1}(F)) \subseteq N$ example $\left(f^{-1}(F)\right) \subseteq N$ for $\left(f^{-1}(F)\right) \subseteq N$. $\left|\frac{1}{r}(F)\right| \subseteq N$ example $Cl(F) = F$. \mathbb{R}^d set in \mathbb{R}^d in \mathbb{R}^d .
 \mathbb{R}^d *eugrbCl* $\left[f(f^{-1}(F)) \right] \subseteq \mathbb{R}$ *eugrbCl* $(F) = F$.

Then N _{eu}
$$
\text{grtCl}\left[f\left(f^{-1}(F)\right)\right] \subseteq N_{eu}
$$
 $\text{grtCl}(F) = F.$
\nBy assumption, assumption, assumption, equations
\n
$$
f\left[N_{eu}\text{grtCl}\left(f^{-1}(F)\right)\right] \subseteq N_{eu}
$$
 $\text{grtCl}\left[f\left(f^{-1}(F)\right)\right]$ neutross
\n $\subseteq F$ $\text{grt} \sim \text{co}$

 $\subseteq F$
and hence $N_{eu} \text{grbCl}[f^{-1}(F)] \subseteq f^{-1}(F)$. Since d hence $N_{eu} \text{grtCl}[f'(F)] \subseteq$
 $T^{-1}(F) \subseteq N_{eu} \text{grtCl}[f^{-1}(F)],$ $f^{-1}(F) \subseteq N$ *eu grb* $Cl[f^{-1}(F)]$,
 N *eu grb* $Cl[f^{-1}(F)] = f^{-1}(F)$. This implies that

 $f^{-1}(F)$ is a neutrosophic grb-closed set in X.

Thus by Theorem 3.4, *f* is a neutrosophic *grb***-**continuous mapping.

Theorem 3.6. Let (X, T_N) and (Y, σ_N) be two neutrosophic topological spaces and $f: X \to Y$ be a mapping. Then *f* is a neutrosophic *grb***-**continuous mapping if and only if σ ² σ - σ for every neutrosophic set *B* in *Y*.

Proof. Let *B* be any neutrosophic set in *Y* and *f* be a neutrosophic *grb***-**continuous mapping. Clearly eutrosophic *grb*-continuous map
 $f^{-1}(B) \subseteq f^{-1}[\N_{eu} \text{grtCl}(B)].$ Then, rosophic *grb*-continuous mapping.
 $(B) \subseteq f^{-1}[\text{N}_{eu} gxdCl(B)].$ Then,
 $(B)] \subseteq \text{N}_{eu} gxdCl[f^{-1}(\text{N}_{eu} gxdCl(B))].$ Clearly $f^{-1}(B) \subseteq f^{-1}[\mathbb{N}_{eu} \text{grdCl}(B)].$ Then,
 $\mathbb{N}_{eu} \text{grdCl}[f^{-1}(B)] \subseteq \mathbb{N}_{eu} \text{grdCl}[f^{-1}(\mathbb{N}_{eu} \text{grdCl}(B))].$ *eu* Since N *eugrbCl B* is neutrosophic *grb***-**closed set in *Y*. So by Theorem 3.4, $f^{-1}\Big[\, \mathbb{N} \,$ _{eu} grb $Cl(B) \Big]$ is a neutrosophic *grb***-**closed set in *X*. Thus, $\begin{aligned} (B) \Big] &\qquad \text{is} \ &\qquad \text{in} \ &\qquad \text{(}B) \Big] \subseteq \ \end{aligned}$ in X . Thus,
 (B)] \subseteq
 $\left(\mathrm{N}_{eu}\text{grtCl}(B)\right)$] = f^{-1} $\left[\mathrm{N}_{eu}\text{grtCl}(B)\right]$. 1 $\begin{aligned} &\mathit{ex}_{\mathit{eu}}\mathit{grt} \mathcal{C}l\Big[f^{-1}\big(B\big)\Big]\subseteq \ &\mathit{ex}_{\mathit{eu}}\mathit{grt} \mathcal{C}l\Big[f^{-1}\Big(\hbox{N}_{\mathit{eu}}\mathit{grt} \mathcal{C}l\big(B\big)\Big)\Big] = f^{-1}\Big[\hbox{N}_{\mathit{eu}}\mathit{grt} \mathcal{C}l\big(B\big)\Big]. \end{aligned}$ *b*-closed set
 e^{ax}
 e^{ax} *grbCl* $\left[f^{-1} (B) \right]$ \overline{a} (B)] =
 $f^{-1}(N \text{ or } tCl(B))$] = $f^{-1}[N]$ \det in
 $\left[f^{-1}(B) \right] \subseteq$ $\begin{bmatrix} f^{-1}(B) \end{bmatrix} \subseteq \ \begin{bmatrix} f^{-1}(\mathbb N_{eu} \text{gr\textbf{t}} Cl(B)) \end{bmatrix} = f^{-1} \begin{bmatrix} \mathbb N_{eu} \text{gr\textbf{t}} Cl(B) \end{bmatrix}.$ N *grb* N _{eu} grb $Cl[f^{-1}(B)] \subseteq$
 N _{eu} grb $Cl[f^{-1}(N_{eu}$ grb $Cl(B))] = f^{-1}[N_{eu}$ grb

Conversely, Conversely,
 N_{eu} *grbCl* $[f^{-1}(B)] \subseteq f^{-1}$ $[N_{eu}$ *grbCl* (B) $]$ for a for all neutrosophic sets B in Y . Let F be a neutrosophic closed set in *Y*. Since every neutrosophic closed set is neutrosophic *grb***-**closed set, 7. Since every neutrosophic closed set
eutrosophic *grb*-closed set,
 $[1(F)] \subseteq f^{-1}[\text{N}_{eu} \text{grtCl}(F)] = f^{-1}(F).$ $\mathbb{E}^{-1}(F)\rbrack \subseteq f^{-1}\lbrack N_{eu}\text{grkCl}(F)\rbrack = f^{-1}(F).$ Fraction Section 1. Since every neutrosophic closed set

s neutrosophic *grb*-closed set,
 $N_{eu} \text{grd}Cl[f^{-1}(F)] \subseteq f^{-1}[\text{N}_{eu} \text{grd}Cl(F)] = f^{-1}(F)$ This implies that $f^{-1}\big(F\big)$ is a neutrosophic *grb***-**closed set in *X*. Thus by Theorem 3.4, *f* is a neutrosophic *grb***-**continuous mapping.

Theorem 3.7. Let (X, T_N) and (Y, σ_N) be two neutrosophic topological spaces and $f: X \rightarrow Y$ be a bijective mapping. Then f is neutrosophic *grb***-**continuous if and only if $\forall x$ a cigence inapping. Then *f* is nearessepine
 $\forall x$ continuous if and only
 $\forall x$ $\exists y$ and only
 $\forall x$ $\exists y$ and only
 $\forall x$ $\exists y$ for every neutrosophic set *A* in *X*.

Proof. Let A be any neutrosophic set in X and f be a bijective and neutrosophic *grb***-**continuous mapping. Let $f(A) = B$. Clearly

Y.

 $f^{-1} \Big[\mathbb{N}_{eu} \text{grd} \text{Int}(B) \Big] \subseteq f^{-1}(B)$. Since *f* an injective mapping, $f^{-1}(B) = A$, so that f^{-1} $\left[\mathbb{N}_{eu}$ *grb* $Int(B)\right] \subseteq A$. Therefore, apping, $f(b) = A$, so that notice
 $h(t(B)) \subseteq A$. Therefore, $N_{eu}g$
 $\left[\left(\mathbb{N}_{eu} g t \Delta t nt(B)\right)\right] \subseteq \mathbb{N}_{eu} g t \Delta t nt(A)$. $\left[\int_{-\infty}^{\infty} \mathbb{N}_{eu} f(t) dt\right]$ $\left[\bigwedge_{e_1} \mathrm{grad}ht(B)\big)\right] \subseteq \mathrm{N}_{e_1}$ grid $Int(A)$. f^{-1} $\left[\begin{array}{c} \mathbb{N} \\ \infty \end{array} \right] \subseteq A$. Therefore,
 \mathbb{N} $_{eu}$ grb $Int\left[f^{-1}\left(\begin{array}{c} \mathbb{N} \\ \infty \end{array} \right] \subseteq \mathbb{N} \right] \subseteq \mathbb{N}$ $_{eu}$ grb $Int(A)$ Since *f* is neutrosophic grb-continuous, f^{-1} $\left[N_{eu}$ grb $Int(B) \right]$ is neutrosophic *grb***-**open set in *X* and utrosophic *grb*-open set in
 $\left[\begin{array}{cc} \mathbb{1} & \mathbb{1} \\ \mathbb{1} & \mathbb{1} \end{array}\right] \subseteq \mathbb{N}_{eu}$ grb $Int(A)$, *eutrosophic grb*-open set
 f^{-1} [N _{eu} grb*atht*(*B*)] \subseteq N _{eu} grb*atht* (*A* N $_{eu}$ grid $Int(B)$ \subseteq N $_{eu}$ grid $Int(A)$,
 $^{-1}$ (N $_{eu}$ grid $Int(B)$) \subseteq f $\big[$ N $_{eu}$ grid $Int(A)$. *eu eu f f Int B f Int A* N N *grb grb* Thus we obtain N_{eu} *grbInt* $[f(A)] \subseteq$ $f\Big[\mathbb{N}\Big]_{eu}$ grb \mathcal{V} nt $\big(A\big)\Big].$ Conversely, Conversely,
 N_{eu} *grbInt* $[f(A)] \subseteq f[N_{eu}$ *grbInt* (A) for example. every neutrosophic set *A* in *X*. Let *V* be a neutrosophic

open set in *Y*. Then *V* is neutrosophic *grb*-open set
in *Y*. Since *f* is surjective and so

$$
V = N_{eu} grbInt(V) = N_{eu} grbInt[f(f^{-1}(V))]
$$
It

$$
\subseteq f[N_{eu} grbInt(f^{-1}(V))].
$$

follows that $f^{-1}(V) \subseteq \mathbb{N}$ eughborhology \mathbb{N} and \mathbb{N} \mathbb{N} is grid \mathbb{N} \mathbb{N} \mathbb{N} and \mathbb{N} \mathbb{N} Therefore $f^{-1}(V)$ is neutrosophic gxb-open set in *X*. Hence *f* a neutrosophic *grb***-**continuous mapping.

Theorem 3.8. Let (X, T_N) and (Y, σ_N) be two neutrosophic topological spaces and $f: X \to Y$ be a mapping. Then *f* is a neutrosophic *grb***-**continuous mapping if and only if grb-continuous mapping if and only if
 $f^{-1} \lceil N_{eu} g x \cdot dnt(B) \rceil \le N_{eu} g x \cdot dnt \lceil f^{-1}(B) \rceil$ for for every neutrosophic set *B* in *Y*.

Proof. Let *B* be any neutrosophic set in *Y* and *f* be a neutrosophic *grb***-**continuous mapping. *f* be a neutrosophic *grb*-continuous mapping.
Clearly $f^{-1}[\N_{eu} gxdnt(t B)] \subseteq f^{-1}$ $\hat{\mathcal{B}}$ implies $\begin{aligned} &\left\lfloor \text{N}_{\text{ eu}} \text{grd} \text{Int}(B) \right\rfloor \subseteq f^{-1} \ &\left(\text{N}_{\text{ eu}} \text{grd} \text{Int}(B) \right) \right] \subseteq \end{aligned}$ $\begin{aligned} &^{1}\!\left(\mathbb{N}_{eu}\mathcal{G}\!\mathcal{I}\!\mathcal{G}\!I\!m\right.\ &^{1}\!\left(B\right)\Big]. \end{aligned}$ \mathcal{L}_{eu} g*x* \mathcal{L}_{eu} for \mathcal{L}_{eu} \mathcal{L}_{eu} for \mathcal{L}_{eu} \mathcal{L}_{eu} for \mathcal eu grì $\frac{1}{2}$ nt $\left[f^{-1}\left(\frac{N}{2}e^{u}\right)\right]$ eu grì $\frac{1}{2}$ nt $\left(B\right)$ \overline{a} \overline{a} $\bigcup_{eu} \mathcal{G} \triangle \mathcal{G}$ N_{eu} grb $Int[f^{-1}(B)]$.
 N_{eu} grb $Int[f^{-1}(B)]$.

Since N_{eu} *grbInt* (B) is neutrosophic *grb*-open set in *Y* and *f* neutrosophic

grb-continuous, f^{-1} $\left[N_{eu}$ grb $Int(B) \right]$ is neutrosophic *grb***-**open set in *X*. Thus grb-open set in
 $(\mathbb{N}_{eu}$ grb $Int(B))] \subseteq$ -1 atrosophic *grb*-open set in
 $\text{curl}\left[f^{-1}\left(\mathbb{N}_{eu}\text{grdM}(B)\right)\right]\subseteq$ neutrosophic *grb*-open set in *X*. Thus
 $N_{eu} g r \Delta t nt \Big[f^{-1} \Big(N_{eu} g r \Delta t nt \Big(B \Big) \Big] \Big] \subseteq$
 $f^{-1} \Big[N_{eu} g r \Delta t nt \Big(B \Big) \Big] \subseteq N_{eu} g r \Delta t nt \Big[f^{-1} \Big(B \Big) \Big].$ $\begin{aligned} \mathbb{E}_{\mathit{eu}} \mathit{grdMt} \Big[\, & f^{-1} \big(\, \mathbb{N} \, \, {_{eu}} \mathit{grdMt} \big(\, B \big) \big) \Big] \subseteq \ & f^{-1} \Big[\, \mathbb{N} \, \, {_{eu}} \mathit{grdMt} \big(\, B \big) \Big] \subseteq \mathbb{N} \, \, {_{eu}} \mathit{grdMt} \Big[\, f^{-1} \big(\, B \big) \Big]. \end{aligned}$ Conversely, $\begin{aligned} \n\int_{0}^{\infty} \left[\int_{0}^{\infty} e^{u} \mathcal{L}^{n} H(u) \right] &\subseteq \mathbb{N} \int_{\text{eu}} \mathcal{L}^{n} H(u) \text{ for } \\ \n\int_{0}^{1} \left[\mathbb{N} \int_{\text{eu}} \mathcal{L}^{n} H(u) \right] &\subseteq \mathbb{N} \int_{\text{eu}} \mathcal{L}^{n} H(u) \text{ for } \\ \n\int_{0}^{1} \left[\mathcal{L}^{n} H(u) \right] &\subseteq \mathbb{N} \int_{\text{eu}} \mathcal{L}^{n} H(u$ for every neutrosophic set B in Y . Let G be any neutrosophic open set in *Y*. Then very neutrosophic set *B* in *Y*. Let *G* be any

neutrosophic open set in *Y*. Then
 $f^{-1}(G) = f^{-1} \Big[N_{eu} g x b t t (G) \Big] \subseteq N_{eu} g x b t t \Big[f^{-1}(G) \Big]$ $f^{-1}(G) = f^{-1}[\text{N}_{eu} \text{grd}nt(G)] \subseteq \text{N}_{eu} \text{grd}nt[f^{-1}(G)]$
and therefore $f^{-1}(G) = \text{N}_{eu} \text{grd}nt[f^{-1}(G)]$. This

implies that $f^{-1}(G)$ is neutrosophic grb -open set in *X*. Hence *f* a neutrosophic *grb***-**continuous mapping.

Theorem 3.9. Let (X, T_N) and (Y, σ_N) be two neutrosophic topological spaces and $f: X \to Y$ be a bijective mapping. Then f is a neutrosophic *grb***-**continuous mapping if and only if *grb*-continuous mapping if and only if $f\left[N_{eu}g\right]\uparrow F(r(A)) \subseteq N_{eu}g\uparrow F\left[f(A)\right]$ for every neutrosophic set *A* in *X*.

Proof. Let f be a bijective and neutrosophic *grb***-**continuous mapping. Let *A* be a neutrosophic set in *X*. By definition, muous mapping. Let *A* be a neutrosophic
in X. By definition,
 $(A) = N_{eu} gr\mathcal{L}Cl(A) \cap N_{eu} gr\mathcal{L}Cl(A^c)$. External of the continuous image grb X . By defining the continuous distribution of the continuous distribution of X . By defining \mathbb{R} and \mathbb{R} and \mathbb{R} \mathbb{R} and \mathbb{R} \mathbb{R} and \mathbb{R} \mathbb{R} By Theorem 3.7, N_{eu} *grb* $Int[f(A)] \subseteq f[N_{eu}$ *grb* $Int(A)]$ and $f[N_{eu}$ *grb* $Int(A)]$ from Theorem 3.5, eu g \exists an [j (11)] \equiv j [\exists eu g \equiv an (11)] \exists and \exists [\exists] \exists [\exists [\exists [\exists [\exists] \exists Fheorem
 $f\left[N_{eu}\text{grd}Cl(A)\right] \subseteq N_{eu}\text{grd}Cl\left[f(A)\right],$
 $f\left[N_{eu}\text{grd}Fr(A)\right] =$

$$
f\Big[\mathbb{N}_{eu}\text{grd}Fr(A)\Big] =
$$
\n
$$
f\Big[\mathbb{N}_{eu}\text{grd}Cl(A)\Big]\cap f\Big[\mathbb{N}_{eu}\text{grd}Cl(A^c)\Big] \subseteq
$$
\n
$$
\mathbb{N}_{eu}\text{grd}Cl\Big[f(A)\Big]\cap \mathbb{N}_{eu}\text{grd}Cl\Big[f(A)\Big]^c
$$
\n
$$
= \mathbb{N}_{eu}\text{grd}Fr\Big[f(A)\Big].
$$

Conversely,

 $f\left[N_{eu}\text{grd}Fr(A)\right]\subseteq N_{eu}\text{grd}Fr\left[f(A)\right]$ for
neutrosophic set *A* in *X*. every neutrosophic *A* in *X*. Then $Tr(A)$ \subseteq N $_{eu}$ grb $Fr[f(A)]$ for every
 $:=$ set A in X. Then
 $F[A] = F[\text{N}_{eu}$ grb $Int(A)] \cup F[\text{N}_{eu}$ grb $Fr(A)]$ rosophic set *A* in *X*. Then
 $I_{eu} \text{grd}Cl(A)$] = $f[N_{eu} \text{grd}Hn(tA)] \cup f[N_{eu} \text{grd}Fr(A)]$
 $(A) \cup N_{eu} \text{grd}Fr[f(A)] \subseteq N_{eu} \text{grd}Cl[f(A)].$ **eutrosophic** set *A* in *X*. Then
 $f\left[N_{eu} \text{grd}Cl(A)\right] = f\left[N_{eu} \text{grd}Mn(A)\right] \cup f\left[N_{eu} \text{grd}Fr(A)\right]$ $_{eu}$ grb $Fr\big[\, f\big(A\big)\big]\subseteq$ N $_{eu}$ grb $\left[\begin{array}{c}\mathbb{N}\end{array}\right]$ *eu grid A* $f(A)\right]$ \cup $f\left[\begin{array}{c}\mathbb{N}\end{array}\right]$ \in $f(A)\cup\mathbb{N}\right]$ \subseteq $\mathbb{N}\left[\begin{array}{c}\text{and }f\end{array}\right]$ \subseteq $\mathbb{N}\left[\begin{array}{c}\text{and }f\end{array}\right]$ $f(A)$ utrosophic set A in X. Then
 $\left[\begin{array}{cc} \text{N}_{eu} g \text{d}C l(A) \end{array}\right] = f \left[\begin{array}{cc} \text{N}_{eu} g \text{d}t n (A) \end{array}\right] \cup f \left[\begin{array}{cc} \text{N}_{eu} g \text{d}t r(A) \end{array}\right]$ $f[N_{eu} g \text{rk} Cl(A)] = f[N_{eu} g \text{rk} Im(A)] \cup f[N_{eu} g \text{rk} Pr(A)]$
 $\subseteq f(A) \cup N_{eu} g \text{rk} Pr[f(A)] \subseteq N_{eu} g \text{rk} Cl[f(A)].$ trosophic set A in X.
 $N_{eu} g \nexists \mathcal{L}l(A) = f \left[N_{eu} g \nexists h t(A) \right] \cup f \left[N \right]$ $\text{grad} f(I(A)) = f[N_{eu} \text{grad} ht(A)] \cup f[N_{eu} \text{grad} f$
) \cup N $_{eu} \text{grad} Fr[f(A)] \subseteq N_{eu} \text{grad} Cl[f(A)]$.

By Theorem 3.5, *f* a neutrosophic *grb***-**continuous mapping.

Theorem 3.10. Let (X, T_N) and (Y, σ_N) be two neutrosophic topological spaces and $f: X \to Y$ be a bijective mapping. Then f is a neutrosophic *grb***-**continuous mapping if and only if 1 1 N N *eugrbFr f B f Fr B eugrb* for every neutrosophic set *B* in *Y*.

Proof. Let f be a bijective and neutrosophic *grb***-**continuous mapping. Let *B* be a neutrosophic set *Y*. By Theorem 3.6, grb-continuous mapping. Let *B* be a neutrosophic
set in *Y*. By Theorem 3.6
 $N_{eu} g r \mathcal{L}Cl[f^{-1}(B)] \subseteq f^{-1}[\text{N}_{eu} g r \mathcal{L}Cl(B)]$. S So f^{-1} [N $_{eu}$ grb**F**r(B)] = rzkCl $[f^{-1}(B)] \subseteq f^{-1}$ N $_{eu}$ grzkCl (B)]. So

N $_{eu}$ grzk $F(R)$] =
 $(N_{eu}$ grzkCl $(B))$ | N $_{eu}$ grzkCl (B^C)] = $[L(B)]$ and $1/(N_{\rm M} \cdot \text{cm}CI(D)) \cap N_{\rm M} \cdot \text{cm}CI(D^{\rm C})$ $\begin{bmatrix} 1 \end{bmatrix}$ and $\begin{bmatrix} 0 \end{bmatrix}$ and $\begin{bmatrix} 0 \end{bmatrix}$ and $\begin{bmatrix} 0 \end{bmatrix}$ $f^{-1}\Big[$ N $_{eu}$ gxt $\mathcal{E}l\big(B\big)\Big]$ =
 $f^{-1}\Big[\Big(\text{N}\big(\begin{matrix}x\end{matrix}\big)$ gxt $\mathcal{L}l\big(B\big)\big)$ \bigcap N $_{eu}$ gxt $\mathcal{L}l\big(B\big)$ $\int f^{-1} \Big[\big(\mathbb{N}_{eu} \text{gr\textsc{t}xCl}(B) \big) \big(\big) \mathbb{N}_{eu} \text{gr\textsc{t}xCl}\big(B^{\text{c}} \big) \Big] \ \int f^{-1} \Big[\mathbb{N}_{eu} \text{gr\textsc{t}xCl}\big(B \big) \Big] \ \int f^{-1} \Big[\mathbb{N}_{eu} \text{gr\textsc{t}xCl}\big(B \big) \Big]$ \overline{a} $\begin{bmatrix} 1 & e^{u} & \cdots & e^{u} \end{bmatrix}$ $\begin{bmatrix} N_{eu} \text{grd}Fr(B) \end{bmatrix} = \begin{bmatrix} N_{eu} \text{grd}Cl(B) \end{bmatrix} \begin{bmatrix} N_{eu} \text{grd}Cl(B^c) \end{bmatrix} =$ $[(N_{eu}gr\mathcal{L}Cl(B))\cap N_{eu}gr\mathcal{L}Cl(B^{c})]=\n\begin{bmatrix}\nN_{eu}gr\mathcal{L}Cl(B)\n\end{bmatrix}\cap f^{-1}[\nN_{eu}gr\mathcal{L}Cl(B^{c})]\n\supseteq\n\begin{bmatrix}\nN_{eu}gr\mathcal{L}Cl(B^{c})\n\end{bmatrix}$ that
 (B) $\bigcap f^{-1}\bigg[N_{eu} g x k C l \bigg(B^C \bigg) \bigg] \supseteq$ that
 (B) $\bigcap f^{-1}\bigg[N_{eu} g x k C l \bigg[f^{-1}\bigg(B^C \bigg) \bigg] =$ neut (B) $\left[\begin{array}{cc} \left(B\right) \end{array}\right] \cap N_{eu}$ grk $Cl\left[f^{-1}\left(B^{C}\right) \right] =$ neutron ne $\begin{bmatrix} \begin{bmatrix} 1 & B \end{bmatrix} \begin{bmatrix} 0 & B \end{bmatrix} \end{bmatrix}$ $\begin{bmatrix} 1(R) \end{bmatrix}$ $\begin{bmatrix} 0 \end{bmatrix}$ $\begin{align} &\int_{\mathcal{C}}\ln\int_{\mathcal{C}}\exp\mathcal{C}U(B)\bigcap\int_{\mathcal{C}}\ln\int_{\mathcal{C}}\exp\mathcal{C}U\Big[|f^{-1}(B)\Big]\bigcap\mathcal{C}\ln\int_{\mathcal{C}}\exp\mathcal{C}U\Big[|f^{-1}(B)\Big]. \end{align}$ $\begin{aligned} \mathcal{L}_{eu} &\text{grad} \left[J_{\perp}\left(B\right) \right] \cap \mathbb{N}_{eu} \text{grad} \left[J_{\perp}\left(B\right) \right] \ &\text{grad} \mathcal{L}l \biggl[f^{-1}\bigl(B\bigr) \biggr] \cap \mathbb{N}_{eu} \text{grad} \mathcal{L}l \biggl[\bigl(f^{-1}\bigl(B\right) \biggr] \end{aligned}$ $\begin{bmatrix} \begin{bmatrix} 1 & 0 \end{bmatrix} & \begin{bmatrix} 1 & 0 \end{bmatrix} & \begin{bmatrix} 1 & 0 \end{bmatrix} \\ \begin{bmatrix} 1 & 0 \end{bmatrix} & \begin{bmatrix} 1 & 0 \end{bmatrix} & \begin{bmatrix} 1 & 0 \end{bmatrix} \end{bmatrix} \end{bmatrix}$ $\begin{bmatrix} -1(R) \end{bmatrix}$ $\begin{bmatrix} 0 \end{bmatrix}$ $\begin{bmatrix} 0 \end{bmatrix}$ $\begin{bmatrix} 0 \end{bmatrix}$ $f^{-1}\Big[\mathbb{N}\Big]_{eu}$ grb $Cl(B)\Big]\bigcap f^{-1}\Big[\mathbb{N}\Big]_{eu}$ grb $Cl\Big(B^C\Big)\Big]$ \sup N $_{eu}$ grb $Cl\Big[f^{-1}\big(B\big)\Big]\bigcap\mathbb{N}\Big]_{eu}$ grb $Cl\Big[f^{-1}\Big(B^C\Big)\Big]$ = N _{eu} grb $Cl[f^{-1}(B)] \cap N$ _{eu} grb $Cl[f^{-1}(B^C)] = N$ ne
 N _{eu} grb $Cl[f^{-1}(B)] \cap N$ _{eu} grb $Cl[(f^{-1}(B))^C]$ N in $=$ N $_{eu}$ grbFr $\left[f^{-1}(B)\right]$. Therefore $=$ N $_{eu}$ grbFr $\Big[f^{-1}(B) \Big]$. Therefore

N $_{eu}$ grbFr $\Big[f^{-1}(B) \Big]$ \subseteq f^{-1} $\Big[$ N $_{eu}$ grbFr (B) $\Big]$. Conversely since S The $\lim_{\epsilon u \to 0} \frac{1}{\epsilon} \int_{\epsilon u}^{\epsilon u} \left[\int_{\epsilon u}^{\epsilon u} \left(\frac{\epsilon u}{\epsilon} \right) \right] \leq \frac{1}{\epsilon} \lim_{\epsilon u \to 0} \frac{\epsilon u}{\epsilon} \frac{\epsilon u}{\epsilon}$ incorrectly since $\lim_{\epsilon u \to 0} \frac{\epsilon u}{\epsilon} \frac{\epsilon u}{\epsilon}$ for every neutrosophic set *B* in *Y*. This implies that every neutrosophic set *B* in *Y*. This implies the N_{eu} grb $Cl[f^{-1}(B)] \subseteq f^{-1}[\text{N}_{eu}$ grb $Cl(B)]$. By Theorem 3.6, *f* is a neutrosophic *grb***-**continuous mapping. *grignal* f is a neutrosophic (ii)
ing.
(X, T_N) and (Y, σ_N) be two ever
graphing if and only if there
 f^{-1} [N_{au}gzzhFr(B)] for ever
primes in Y .
bijective and neutrosophic field bijective and neutrosophic field bi

Definition 3.11. Let $x_{(r,t,s)}$ be a neutrosophic point of a neutrosophic topological space (X, T_N) . A neutrosophic set *A* of *X* is called neutrosophic neighbourhood of $x_{(r,t,s)}$ if there exists a neutrosophic open set *B* such that $x_{(r,t,s)} \in B \subseteq A$.

Theorem 3.12. Let *f* be a mapping from a neutrosophic topological space (X, T_N) to a neutrosophic topological space (Y, σ_N) . Then the following assertions are equivalent.

 (i) *f* is neutrosophic

For each neutrosophic point $x_{(r,t,s)} \in X$ and every neutrosophic neighbourhood A of $f(x_(r,t,s))$, there exists a neutrosophic *grb***-**open set *B* such that $x_{(r,t,s)} \in B \subseteq f^{-1}(A)$.

(iii) For each neutrosophic point $x_{(r,t,s)} \in X$ and every neutrosophic neighbourhood *A* of $f\left(x_{(r,t,s)}\right)$, there exists a neutrosophic grb-open set *B* in *X* such that $x_{(r,t,s)} \in B$ and $f(B) \subseteq A$.

Proof. (i) \Rightarrow (ii): Let $x_{(r,t,s)} \in X$ be a neutrosophic point in X and let A be a neutrosophic neighbourhood of $f(x_(r,t,s))$. Then there exists a neutrosophic open set B in Y such that $f\left(x_{(r,t,s)}\right) \in B \subseteq A$. Since f is neutrosophic grb-continuous, we know that $f^{-1}(B)$ is

neutrosophic grb-open set in X
 $x_{(r,t,s)} \in f^{-1}(f_{(r,t,s)}) \subseteq f^{-1}(B) \subseteq f^{-1}(A)$. is a neutrosophic *grb***-**open set in *X* and $(f_{(r,t,s)}) \in f^{-1}(f_{(r,t,s)}) \subseteq f^{-1}(B) \subseteq f^{-1}(A).$ reutrosophic grb-open set i
 $x_{(r,t,s)} \in f^{-1}(f_{(r,t,s)}) \subseteq f^{-1}(B) \subseteq f^{-1}(A)$ This implies (ii) is true.

 $(ii) \Rightarrow (iii)$: Let $x_{(r,t,s)}$ be a neutrosophic point in *X* and let *A* be a neutrosophic neighbourhood of $f\left(x_{(r,t,s)}\right)$. The condition (ii) implies that there exists a neutrosophic ϵ *grb*-open set *B* in X such that $x_{(r,t,s)} \in B \subseteq f^{-1}(A)$. Thus $x_{(r,t,s)} \in B$ and $f(B) \subseteq f[f^{-1}(A)] \subseteq A$. Hence (iii) is true.

 $(iii) \Rightarrow (i)$: Let *B* be a neutrosophic open set in *Y* and let $x_{(r,t,s)} \in f^{-1}(B)$. Since *B* is neutrosophic open set, $f\left(x_{(r,t,s)}\right) \in B$, and so *B* is neutrosophic neighbourhood of $f(x_(r,t,s))$. It follows from (iii) that there exists a neutrosophic *grb***-**open set *A* in *X* such that $x_{(r,t,s)} \in A$ and $f(A) \subseteq B$ so that X such that $x_{(r,t,s)} \in A$ and $f(A) \subseteq B$ so that
 $x_{(r,t,s)} \in A \subseteq f^{-1}[f(A)] \subseteq f^{-1}(B)$. This implies by definition that $f^{-1}(B)$ is a neutrosophic α -open set in X. Therefore, f is a neutrosophic *grb***-**continuous mapping.

IV. NEUTROSOPHIC grb-IRRESOLUTE MAPPINGS

In this section, we introduce the concept of neutrosophic *grb***-**irresolute mappings in neutrosophic topological spaces. Also, we discuss the relationship with neutrosophic *grb***-**continuous mappings.

Definition 4.1. Let (X, T_N) and (Y, σ_N) be two neutrosophic topological spaces. A mapping $f: X \rightarrow Y$ is called neutrosophic *grb*-irresolute if the inverse image of every neutrosophic *grb***-**open set in *Y* is neutrosophic *grb***-**open in *X*. **Theorem 4.2.** Let (X, T_N) and (Y, σ_N) be two neutrosophic topological spaces. A mapping $f: X \rightarrow Y$ is called neutrosophic *grb*-irresolute if the inverse image of every neutrosophic *grb***-**closed set in *Y* is neutrosophic *grb***-**closed in *X*. **FROSPHIC** grb-intensity in the continuous continuous.
 Theorem 4.4.
 Continuous continuous. Theorem of mapping The content of equivalent:
 grb-intensity in population mapping in the content of the property neutroso

Proof. Let *A* be any neutrosophic *grb***-**closed set in *Y*. Then A^C is neutrosophic gxb-open set in *Y*. Since f is neutrosophic grb-irresolute, $f^{-1}(A^c)$ is neutrosophic *grb***-**open set in *X* and $f^{-1}(A^c) = [f^{-1}(A)]^c$ which implies that $f^{-1}(A)$ is neutrosophic *grb***-**closed set in *X*.

Conversely, Let *B* be any neutrosophic α *grb***-**open set in *Y*. Then *B^c* is neutrosophic *grb***-**closed set in *Y*. Thus $f^{-1}(B^C)$ is neutrosophic *grb***-**closed set in *X* and $f^{-1}(B^c) = [f^{-1}(B)]^c$ which implies that $f^{-1}(B)$ is neutrosophic *grb***-**open set in *X*. Hence $f: X \rightarrow Y$ is neutrosophic gxb-irresolute.

Theorem 4.3. Every neutrosophic *grb***-**irresolute mapping is neutrosophic *grb***-**continuous.

Proof. Let *V* be a neutrosophic open set in *Y*. Since every neutrosophic open set is neutrosophic *grb***-**open, *V* is neutrosophic *grb***-**open. Since *f* is neutrosophic *grb*-irresolute, $f^{-1}(V)$ $f^{-1}(V)$ is neutrosophic *grb***-**open in *X*. Therefore *f* is neutrosophic *qrb*-continuous.

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Let $f:(X,T_N) \to (Y,\sigma_N)$ be a mapping. Then the following assertions are equivalent:

(i) f is neutrosophic ϵ *grb*-irresolute.

(i) f is neutrosophic grb-irresolute.

(ii) N_{eu} grbcl[$f^{-1}(B)$] $\subseteq f^{-1}$ [N_{eu} grbcl(B)] for for every neutrosophic set *B* of *Y*.

very neutrosophic set *B* of *Y*.

(iii) $f[N_{eu} \text{ gridCl}(A)] \subseteq N_{eu} \text{ gridCl}[f(A)]$ for every neutrosophic set *A* of *X*. the proposition of $\lim_{u \to 0} \int \ln \lim_{u \to 0} \frac{\partial f}{\partial x}$ is $\lim_{u \to 0} \frac{\partial f}{\partial x}$ of X .

(iv) $f^{-1}[\ln \lim_{u \to 0} \frac{\partial f}{\partial x}$ $f(B)] \subseteq \mathbb{N}_{eu}$ $\lim_{u \to 0} \frac{\partial f}{\partial x}$ or

(iv)
$$
f^{-1}[\text{N}_{eu} \text{grd}tnt(B)] \subseteq \text{N}_{eu} \text{grd}tnt[f^{-1}(B)]
$$
 for
every neutrosophic set *B* of *Y*.

Proof. (i) \Rightarrow (ii): Let *B* be any neutrosophic set in *Y*. Then N_{eu} grb $Cl(B)$ is neutrosophic grb-closed set in *Y*. Since *f* is neutrosophic grb**-irresolute**, f^{-1} $\left[N \right]$ *eu grb* $Cl(B)$ is neutrosophic *grb***-**closed set in *X*. Then g*rb*-closed set in *X*. Then
 N_{eu} grb $C[\int f^{-1}(\text{N}_{eu} \text{grd}C(\text{B}))] = f^{-1}[\text{N}_{eu} \text{grd}C(\text{B})]$. Clearly it follows that $N_{eu} \text{ grid } Cl\left[f^{-1}(B)\right] \subseteq$
 $N_{eu} \text{ grid } Cl\left[f^{-1}(N_{eu} \text{ grid } Cl(B))\right] = f^{-1}\left[N_{eu} \text{ grid } Cl(B)\right].$ early it follows that $N_{eu} gxdCl[f^{-1}(B)]g$
 $_{eu} gxdCl[f^{-1}(N_{eu} gxdCl(B))] = f^{-1}[N_{eu} gxdCl(B)].$ This proves (ii) .

 $(ii) \Rightarrow (iii)$: Let *A* be any neutrosophic set in *X*. Then $f(A) \subset Y$. By inen $J(A) \subseteq I$.

(ii), $N_{eu} \text{grd}Cl[f^{-1}(f(A))] \subseteq$
 $f^{-1} \Big[N_{eu} \text{grd}Cl(f(A)) \Big]$.

$$
f^{-1} \Big[N_{eu} \text{grdCl}(f(A)) \Big].
$$
 But

 f^{-1} $\left[N_{eu}$ grb $Cl(f(A)) \right]$.
 N_{eu} grb $Cl(A) \subseteq N_{eu}$ grb $Cl[f^{-1}(f(A))]$, so we obtain N_{eu} grb $Cl(A) \subseteq f^{-1} \left[N_{eu}$ grb $Cl(f(A))\right]$.
Thus $f\left[N_{eu}$ grb $Cl(A) \right]\subseteq N_{eu}$ grb $Cl\left[f(A)\right]$. obtain N_{eu} grb $Cl(A) \subseteq f \cap [N_{eu}$ grb $Cl(f(A))]$
Thus $f[N_{eu}$ grb $Cl(A)] \subseteq N_{eu}$ grb $Cl[f(A)]$.

 $(iii) \Rightarrow (i)$: Let *F* any neutrosophic *grb***-**closed set in *Y*. Then grb-closed set in
 $f^{-1}(F) = f^{-1} \Big[N_{eu} g x k C l(F) \Big].$ By et in *Y*. Then
 $\left[\begin{array}{ccc} \mathbb{N} & \text{if } \mathbb{N} \\ \text{if } \mathbb{N} \end{array}\right]$ $\mathbb{E}\left[\begin{array}{ccc} \mathbb{N} & \text{if } \mathbb{N} \\ \text{if } \mathbb{N} \end{array}\right]$ $\mathbb{E}\left[\begin{array}{ccc} \text{if } \mathbb{N} \end{array}\right]$ $\mathbb{E}\left[\begin{array}{ccc} \text{if } \mathbb{N} \end{array}\right]$ $\mathbb{E}\left[\begin{array}{ccc} \text{if } \mathbb{N} \end{array}\$ $f^{-1}(F) = f^{-1}[\mathbb{N}]$ and $\mathcal{L}l(F)$. By (
 $f\left[\mathbb{N}]$ and $\mathcal{L}l(f^{-1}(F))\right] \subseteq \mathbb{N}$ and $\mathcal{L}l[f(f^{-1}(F))]$
 $\equiv \mathbb{N}]$ and $\mathcal{L}l(F) = F$. N_{eu} grb $Cl(F)$ = F . $\mathcal{C}l(f^{-1}(F))$
 $\mathcal{C}l(F) = F$ ⁻¹ $(F) = f^{-1} \Big[N_{eu}$ gri $Cl(F)$]. By (iii),
 $\Big[N_{eu}$ gri $Cl(f^{-1}(F)) \Big] \subseteq N_{eu}$ gri $Cl[f(f^{-1}(F))]$ $f\left[\begin{array}{l}\mathcal{N}\end{array}_{eu}\right]$ g \mathcal{N} $\mathcal{N}\left(f^{-1}(F)\right)\right]\subseteq$
 \subseteq $\mathcal{N}\left[\mathcal{N}\right]$ \subseteq f $\mathcal{N}\left(F\right)=F$. $(F) = f^{-1} \Big[N_{eu} \text{grtCl}(F) \Big].$
 $N_{eu} \text{grtCl}(f^{-1}(F)) \Big] \subseteq N_{eu} \text{grtCl}$ That implies, \mathbb{N}_{eu} grb $Cl[f^{-1}(F)] \subseteq f^{-1}(F)$. But $f^{-1}(F) \subseteq \mathbb{N}_{eu}$ grb $Cl[f^{-1}(F)]$,

$$
f^{-1}(F) \subseteq \mathbb{N}_{eu} \text{grdCl}[f^{-1}(F)],
$$

$$
\mathbb{N}_{eu} \text{grdCl}[f^{-1}(F)] = f^{-1}(F) \text{ and so } f^{-1}(F) \text{ is}
$$

neutrosophic *grb***-**closed set in *X*. Therefore *f* is neutrosophic *grb***-**irresolute.

 $(i) \Rightarrow (iv)$: Let *B* be any neutrosophic set in *Y*. We know that N_{eu} *grb* $Int(B)$ is neutrosophic *grb***-**open set in *Y*. Since *f* is neutrosophic grb-irresolute, $f^{-1} \left[\mathbb{N}_{eu} \text{grb} \right]$ is neutrosophic $(grb\text{-}open)$
 (B)] = set in *X*. Then $\begin{aligned} &\left(\,B\,\right)\rfloor = \ &\left(\,\mathbb{N}\, \mathrm{ }_{e\mu}\mathcal{G}\vec{x}\vec{b}nt\big(B\big)\big)\right] \subseteq \end{aligned}$ $\begin{aligned} &^{1}\!\left(\mathbb{N}_{eu}\mathcal{G}\!\mathcal{I}\!\mathcal{G}\!I\!n\right)\ &^{1}\!\left(B\right)\end{aligned}$ 1 \mathcal{L}_{eu} g*x* \mathcal{L}_{eu} for \mathcal{L}_{eu} \mathcal{L}_{eu} for \mathcal{L}_{eu} for reutrosophic *gr*
 $f^{-1} \Big[\, \mathbb{N} \, \, {}_{eu}$ *gridnt* (B *eu* gridnt $\left[f^{-1}\left(\frac{N}{e_u}\right)$ gridnt $\left(B\right)$ - \overline{a} \overline{a} rosophic *grb*-open
 $\left[N_{eu} g r \Delta n t (B) \right] =$ $\bigcup_{eu} \text{grad}m(\mathcal{L})\big] =$
N $_{eu} \text{grad}m\big(f^{-1}\big(\mathbb{N}\big)$ $_{eu} \text{grad}m\big(B\big)\big)\big] \subseteq$ N_{eu} grb $Int[f^{-1}(B)]$.
 N_{eu} grb $Int[f^{-1}(B)]$. neutrosophic *grb*--closed set in *x*. Therefore *f* is **Since**

neutrosophic *grb*--inresolute.

(i) \Rightarrow (iv): Let *B* be any neutrosophic set in *Y*. Since

(i) \Rightarrow (iv): Let *B* be any neutrosophic set in *Y*. Since

 $(iv) \Rightarrow (i)$: Let *V* be any neutrosophic *grb*-open set in *Y*. Then by et in *Y*. Then by (iv),
 $f^{-1}(V) = f^{-1} \Big[N_{eu} g x \Delta tnt(V) \Big] \subseteq$
 $N_{eu} g x \Delta tnt \Big[f^{-1}(V) \Big]$. But, N _{eu} grì \mathcal{V} Int $\left[f^{-1}\big(V\big)\right].$ But, $\binom{1}{V}$.
 $\subseteq f^{-1}(V),$ $\lim_{\text{eu}} \text{grd} \text{Int} \left[f^{-1}(V) \right] \subseteq f^{-1}(V),$
 $\lim_{\text{eu}} \text{grd} \text{Int} \left[f^{-1}(V) \right] = f^{-1}(V)$ and hence $f^{-1}(V)$

is neutrosophic σ *zb*-open. Thus *f* is neutrosophic *grb***-**irresolute.

Theorem 4.5. If $f:(X, T_N) \to (Y, \sigma_N)$ and and $g:(Y,\sigma_{N})\rightarrow (Z,\eta_{N})$ are neutrosophic *grb***-**irresolute, their composition $gof:(X, T_N) \rightarrow (Z, \eta_N)$ is also neutrosophic *grb***-**irresolute.

Proof. Let *V* be a neutrosophic *grb*-open set in *Z*. Since *g* is a neutrosophic *grb***-**irresolute mapping, $g^{-1}(V)$ is neutrosophic grb-open in Y. Since *f* is a neutrosophic *grb***-**irresolute mapping, a neutrosophic *gro*-irresort
 $f^{-1}\left[g^{-1}(V)\right] = (gof)^{-1}(V)$ is neutrosophic σ *p* σ **b**-open in X. Therefore $g \circ f$ is neutrosophic *grb***-**irresolute.

Theorem 4.6. If $f: (X, T_N) \to (Y, \sigma_N)$ is neutrosophic *grb***-**irresolute and $g:(Y,\sigma_{N})\rightarrow (Z,\eta_{N})$ is neutrosophic grb-continuous, then their composition $gof:(X,T_N) \rightarrow (Z,\eta_N)$ is also

Let V be a neutrosophic open set in Z . Since *g* is a neutrosophic *grb***-**continuous mapping, $g^{-1}(V)$ is neutrosophic gxb-open set in *Y*. Since *f* is a neutrosophic *grb***-**irresolute mapping, is a neutrosophic gro-fresone
 $f^{-1}\left[g^{-1}(V)\right] = (gof)^{-1}(V)$ is neutrosophic *grb*-open in *X*. Therefore *gof* is neutrosophic *grb***-**conrinuous.

V. NEUTROSOPHIC grb-CLOSED MAPPINGS AND NEUTROSOPHIC grb-OPEN MAPPINGS

In this section, we introduce neutrosophic *grb***-**closed mappings and neutrosophic *grb***-**open mappings in neutrosophic topological spaces and obtain certain characterizations of these classes of mappings.

Definition 5.1. Let (X, T_N) and (Y, σ_N) be two neutrosophic topological spaces. A function $f: (X, T_N) \to (Y, \sigma_N)$ is said to be neutrosophic *grb***-**closed if the image of each neutrosophic closed set in *X* is neutrosophic *grb***-**closed in *Y*.

Definition 5.2. Let (X, T_N) and (Y, σ_N) t be two neutrosophic topological spaces. A function $f: (X, T_N) \to (Y, \sigma_N)$ is said to be neutrosophic *grb***-**open if the image of each neutrosophic open set in *X* is neutrosophic *grb***-**open in *Y*.

Theorem 5.3. A function $f:(X,T_N) \to (Y,\sigma_N)$ is said to be neutrosophic \mathcal{G} *rb*-closed if and only if $N_{eu}\mathcal{G}$ *rt* $Cl[f(A)] \subseteq f[N_{eu}Cl(A)]$ for every N_{eu} *grbCl* $[f(A)] \subseteq f[N_{eu}Cl(A)]$ for every neutrosophic set *A* of *X*.

Proof. Suppose $f: (X, T_N) \to (Y, \sigma_N)$ is a neutrosophic *grb***-**closed function and *A* is any neutrosophic set in X. Then $N_{eu}Cl(A)$ is a neutrosophic closed set in *X*. Since *f* is neutrosophic *grb*-closed, $f[N_{eu}Cl(A)]$ is a neutrosophic *grb***-**closed set in *Y*. Thus neutrosophic *grb*-closed, $f\left[\begin{array}{cc} N_{eu}Cl(A) \end{array}\right]$ is a
neutrosophic *grb*-closed set in *Y*. Thus
N $_{eu} gr\alpha C l \left[f\left(N_{eu}Cl(A)\right)\right] = f\left[\begin{array}{cc} N_{eu}Cl(A) \end{array}\right].$ Therefore N_{eu} *grbCl* $\left[f(A)\right] \subseteq$ Therefore $N_{eu} \text{grbCl}\Big[f\Big(A\Big)\Big] \subseteq$
 $N_{eu} \text{grbCl}\Big[f\Big(A\Big)\Big] \subseteq$
 $N_{eu} \text{grbCl}\Big[f\Big(A\Big)\Big] = f\Big(N_{eu}Cl\Big(A\Big)\Big).$ Hence N _{eu} gric $Cl[f(N_{eu}Cl(A))] = f(N_{eu}Cl(A))$. Hence
 $N_{eu}glCCl[f(A)] \subseteq f(N_{eu}Cl(A))$. N_{ev} *grbCl* $f(A)$ \subseteq $f(N_{ev}Cl(A))$.

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Conversely, let *A* be a neutrosophic closed set in *X*. Then $N_{eu}Cl(A) = A$ and so $f(A) = f \left[\mathbb{N}_{eu} Cl(A) \right].$ our assumption $\lim_{\text{eu}} \text{grdCl}\left[f(A)\right] \subseteq f(A).$ But

 $f(A) \subseteq N$ _{eu} gri $Cl[f(A)]$.
 $f(A)$ Hence N_{eu} grb $Cl[f(A)] = f(A)$ and therefore $f(A)$ is neutrosophic *grb***-**closed set in *Y*. Thus *f* is a neutrosophic *grb***-**closed mapping.

Theorem 5.4. A mapping $f:(X,T_N) \to (Y,\sigma_N)$ is neutrosophic *grb***-**closed if and only if for each neutrosophic set *W* of *Y* and for each neutrosophic open set U of X containing $f^{-1}(W)$ there exists a neutrosophic *grb***-**open set *V* of *Y* such that $W \subseteq V$ and $f^{-1}(V) \subseteq U$.

Proof. Suppose *f* a neutrosophic *grb***-**closed mapping. Let *W* be any neutrosophic set in *Y* and *U* be a neutrosophic *grb***-**open set of *X* such tha $f^{-1}\big(W\bigl)\!\subseteq\! U.$ Then $V = \left[f(U^c) \right]^c$ is neutrosophic *grb***-**open set containing *W* such that $f^{-1}(V) \subseteq U$. Conversely, let *W* be a neutrosophic closed set of *X*. Then $f^{-1}\left[\left(f(W)\right)^c\right] \subseteq W^c$ and W^c is neutrosophic open in *X*. By assumption, there exists a neutrosophic *grb***-**open set *V* of *Y* such that $\left[f \left(W \right) \right]^{c} \subseteq V$ and $f^{-1}(V) \subseteq W^c$ and so $W {\;\subseteq\;} \bigl[\, f^{-1} \bigl(V \bigr) \bigr]^{\!c}$. Hence et *A* be a neutrosophic closed set in **Proof.**
 $\int_{\alpha} G'(A) = A$ and so *X*. Siff $\alpha(T(A)) = B$ and so *X*. Siff $\alpha(T(A)) = \beta(T(A))$. But gentroff $\beta(T(A)) = f(A)$ there neutroff $T(A) = f(A)$ there neutroff $\beta(T(A)) = f(A)$ and therefore $f(A)$ i

 $\begin{aligned} &\left(\begin{matrix} V \end{matrix} \right) \end{aligned}^c.$ Hence
 $\left(W \right) \subseteq f \bigg[\left(f^{-1} \big(V \big) \right)^c \bigg] \subseteq V^c,$ \subseteq $f(W)$ \subseteq f $\left[(f^{-1}(V))^c \right] \subseteq$ V^c *c* $V^c \subseteq f(W) \subseteq f \left[\left(f^{-1}(V) \right)^c \right] \subseteq V^c$, which implies $f(W) = V^c$. Since V^c is neutrosophic grb -closed, $f(W)$ is neutrosophic grb -closed and *f* is neutrosophic *grb***-**closed mapping.

Theorem 5.5. Let $f: (X, T_N) \to (Y, \sigma_N)$ be a neutrosophic closed mapping and $g:(Y, \sigma_N) \to (Z, \eta_N)$ be a neutrosophic *grb***-**closed mapping. Then their composition $gof: (X,T_N) \rightarrow (Z,\eta_N)$ is neutrosophic qxb-closed.

Let *F* be a neutrosophic closed set in *X*. Since f is neutrosophic closed, $f(F)$ is neutrosophic closed in *Y*. Since *g* is neutrosophic

grb-closed, $g[f(F)] = (gof)(F)$ is

neutrosophic $\frac{grb}{dr}$ -closed in Z. Hence $\frac{gof}{dr}$ is a neutrosophic *grb***-**closed mapping.

Theorem 5.6. Let $f: (X, T_{N}) \rightarrow (Y, \sigma_{N})$ and $g:(Y,\sigma_{N})\rightarrow (Z,\eta_{N})$ be two mappings such that their composition $gof: (X,T_N) \rightarrow (Z,\eta_N)$ is neutrosophic *grb***-**closed. Then the following statements are true.

 (i) *f* is neutrosophic continuous and surjective, then *g* is neutrosophic *grb***-**closed.

(ii) If g is neutrosophic g _{rb}-irresolute and injective, then *f* is neutrosophic *grb***-**closed.

Proof. (i) Let A be a neutrosophic closed set of Y. Since $f^{-1}(A)$ neutrosophic continuousis *f* is neutrosophic closed in *X*. Since *g f* is neutrosophic grb-closed, $(gof)(f^{-1}(A))$ is neutrosophic *grb***-**closed in *Z*. Since *f* is surjective, $g(A)$ is neutrosophic $gx + b$ -closed in Z. Hence *g* is neutrosophic *grb***-**closed.

(ii) Let *B* be any neutrosophic closed set of X. Since $g \circ f$ is neutrosophic \mathcal{G} *zb*-closed, $(g \circ f)(B)$ is neutrosophic *grb***-**closed in *Z*. Since *g* is neutrosophic *grb***-**irresolute, ¹ $g^{-1}(gof(B))$ is neutrosophic *grb***-closed** in *Y*. Since *g* is injective, $f(B)$ is **neutrosophic** *grb*-**closed** in *Y*. Hence *f* is neutrosophic *grb***-**closed.

Theorem 5.7. Let $f: (X, T_N) \to (Y, \sigma_N)$ be a neutrosophic *grb***-**closed mapping.

 (i) If A is neutrosophic closed set of X, then the restriction $f_A: A \to Y$ is neutrosophic grb-closed.

(ii) If $A = f^{-1}(B)$ for some neutrosophic closed set B of *Y*, then the restriction $f_A: A \to Y$ is neutrosophic *grb***-**closed.

Proof. (i) Let B be any neutrosophic closed set of *A*. Then $B = A \cap F$ for some neutrosophic closed set F of X and so B is neutrosophic closed in X . By hypothesis, $f(B)$ is neutrosophic grb-closed in *Y*. But $f(B) = f_A(B)$, therefore f_A is a neutrosophic *grb***-**closed mapping.

(ii) Let *D* be any neutrosophic closed set of *A*. Then $D = A \cap H$ for some neutrosophic closed
set *H* in *X*. Now,
 $f_A(D) = f(D) = f(A \cap H) = f[f^{-1}(B) \cap H]$ set *H* in *X*. Now,

set *H* in *X*. Now,
\n
$$
f_A(D) = f(D) = f(A \cap H) = f[f^{-1}(B) \cap H]
$$
\n
$$
= B \cap f(H).
$$

Since *f* is a neutrosophic *grb***-**closed mapping, so $f(H)$ is a neutrosophic gxb-closed set in Y. Hence f_A is a neutrosophic σ *zb*-closed mapping. **Theorem 5.8.** A function $f:(X,T_N) \to (Y,\sigma_N)$ is neutrosophic *grb***-**open if and only if neutrosophic *grb*-open if and o
 $f\left[\begin{array}{cc}N_{eu}Int(A)\end{array}\right] \subseteq N_{eu}$ grid $Int\left[f(A)\right]$, for for every neutrosophic set *A* of *X*.

Proof. Suppose $f:(X,T_N) \to (Y,\sigma_N)$ is a neutrosophic *grb***-**open function and *A* is any neutrosophic set in *X*. Then $N_{eu}Int(A)$ is a neutrosophic open set in X . Since f neutrosophic grb-open, $f[N_{eu}Int(A)]$ is a neutrosophic
 grb-open set. Since
 $N_{eu}grdint[f(N_{eu}IntA)] \subseteq N_{eu}gtdInt[f(A)],$ *grb***-**open Since

$$
\begin{array}{ll}\n\text{grb-open set.} & \text{Since} & \text{grb} \\
\mathbb{N}_{eu}\text{grid}nt[f(\mathbb{N}_{eu}\text{Int}A)] \subseteq \mathbb{N}_{eu}\text{grid}nt[f(A)], & \text{neu} \\
f[\mathbb{N}_{eu}\text{Int}(A)] \subseteq \mathbb{N}_{eu}\text{grid}nt[f(A)]. & \mathcal{Y}_{(k)} \\
\text{Conversely, } f[\mathbb{N}_{eu}\text{Int}(A)] \subseteq \mathbb{N}_{eu}\text{grid}nt[f(A)] \\
\text{for every neutrosophic set } A \text{ in } X. \text{ Let } U \text{ be a}\n\text{neutrosophic open set} & \text{in} & X. \text{ Then} & \text{neu} \\
\mathbb{N}_{eu}\text{Int}(U) = U \text{ and} & \text{by} & \text{hypothesis,} & \text{grb} \\
f(U) \subseteq \mathbb{N}_{eu}\text{grid}nt[f(U)].\text{But} & Y.\n\end{array}
$$

 $U(U) \subseteq N$ *eu* $\mathcal{Q} \subseteq \mathcal{Q}$ *eu* $\mathcal{Q} \subseteq \mathcal{Q}$ (U) . Therefore,

 $f(U) = N_{eu}$ grbth $[f(U)]$. Then $f(U)$ is

neutrosophic *grb***-**open. Hence *f* is a neutrosophic *grb***-**open mapping.

Theorem 5.9. A function $f:(X,T_N) \rightarrow (Y,\sigma_N)$ is neutrosophic *grb***-**open if and only if for each

 $x_{(r,s,t)} \in X$ and for each neutrosophic neighborhood *U* of $x_{(r,s,t)}$ in *X*, there exists a neutrosophic grb-neighborhood *W* of $f(x_{(r,s,t)})$ in *Y* such that $W \subseteq f(U)$.

Proof. Let $f: (X, T_N) \to (Y, \sigma_N)$ be a neutrosophic grb -open function. Let $x_{(r,s,t)} \in X$ and *U* be any arbitrary neutrosophic neighborhood of $x_{(r,s,t)}$ in X. Then there exists a neutrosophic open set *G* such that $x_{(r,s,t)} \in G \subseteq U$. By Theorem open set *G* such that $x_{(r,s,t)} \in G \subseteq U$. By Theorem
5.8, $f(G) = f[\mathbb{N}_{eu} Int(G)] \subseteq \mathbb{N}_{eu} \text{grd} Int[f(G)]$. But, N_{eu} *grb* $Int[f(G)] \subseteq f(G)$. Therefore, N_{eu} grbtnt $[f(G)] = f(G)$ and hence $f(G)$ is neutrosophic grb -open in Y. Since $x_{(r,s,t)} \in G \subseteq U$, *relationsophic gzb*-open in *Y*. Since $x_{(r,s,t)} \in G \subseteq U$,
 $f(x_{(r,s,t)}) \in f(G) \subseteq f(U)$ and so the result follows by taking $W = f(G)$.

Conversely, Let *U* be any neutrosophic open set in X . Let $x_{(r,s,t)} \in U$ and $f(x_{(r,s,t)}) = y_{(k,l,m)}$. Then by assumption there exists a neutrosophic grb-neighborhood $W_{(y_{(k,l,m)})}$ of $y_{(k,l,m)}$ in Y such that $W_{\left(y_{\left(k,l,m\right)}\right)} \subseteq f\left(U\right)$. Since $W_{\left(y_{\left(k,l,m\right)}\right)}$ is a neutrosophic grb **-neighborhood** of $y_{(k,l,m)}$, there exists a neutrosophic *grb***-**open set $V_{(y_{(k,l,m)})}$ in *Y* such that $\in V_{(v, \dots)} \subseteq W_{(v, \dots)}.$ $y_{(k,l,m)} \in V_{(k,l,m)} \subseteq W_l$ Therefore,

$$
y_{(k,l,m)} \in V_{(y_{(k,l,m)})} \subseteq W_{(y_{(k,l,m)})}.
$$
 Therefore,

 $f(U) = \bigcup \{ V_{(k,l,m)} : y_{(k,l,m)} \in f(U) \}$. Since the union of neutrosophic *grb***-**open neutrosophic $\text{grb}-\text{open}, f(U)$ is a neutrosophic $\text{grb}-\text{open}$ set in *Y*. Thus, *f* is a neutrosophic *grb***-**open mapping.

Theorem 5.10. any bijective mapping $f: (X, T_N) \to (Y, \sigma_N)$ the following statements are equivalent:

(i) $f^{-1}: Y \to X$ is neutrosophic grb-continuous.

- (ii) f is neutrosophic \mathcal{G} *rb***-**open.
- (iii) *f* is neutrosophic *grb*-closed.

Proof. (i) \Rightarrow (ii): Let *U* be a neutrosophic open set in *X*. By assumption, $(f^{-1})^{-1}(U) = f(U)$ is $f^{-1}\big)^{-1}(U) = f(U)$ is neutrosophic σ *p* σ **b**-open in *Y* and so *f* is neutrosophic *grb***-**open.

 $(ii) \Rightarrow (iii)$: Let *F* be a neutrosophic closed set of *X*. Then F^c is a neutrosophic open set in *X*. By assumption $f(F^c)$ is neutrosophic grb-open in *Y*. But $f(F^c) = [f(F)]^c$. Therefore $f(F)$ is neutrosophic *grb***-**closed set in *Y*. Hence, *f* is neutrosophic *grb***-**closed.

 $(iii) \Rightarrow (i)$: Let *F* be a neutrosophic closed set of *X*. By assumption, $f(F)$ is neutrosophic g *rb*-closed set in *Y*. But $f(F) = (f^{-1})^{-1}(F)$ and $f(F) = (f^{-1})^{-1}(F)$ and therefore by Theorem 3.4, $f^{-1}: Y \to X$ is neutrosophic grb-continuous.

VI. STRONGLY NEUTROSOPHIC grb-CONTINUOUS AND PERFECTLY grb-CONTINUOUS MAPPINGS

In this section, we introduce and study the concepts of strongly neutrosophic *grb***-**continuous and perfectly neutrosophic *grb***-**continuous mappings in neutrosophic topological spaces.

Definition 6.1. A mapping $f: (X, T_{N}) \rightarrow (Y, \sigma_{N})$ is called strongly neutrosophic *grb***-**continuous if the inverse image of every neutrosophic *grb***-**open set in *Y* is neutrosophic open in *X*.

Definition 6.2. A mapping $f: (X, T_N) \to (Y, \sigma_N)$ is called perfectly neutrosophic *grb***-**continuous if the inverse image of every neutrosophic *grb***-**open set in *Y* is neutrosophic clopen in *X*.

Theorem 6.3. Let $f:(X,T_N) \rightarrow (Y,\sigma_X)$ be a mapping. Then the following statements are true:

(i) If f is perfectly neutrosophic g *rb*-continuous, then f is perfectly neutrosophic continuous.

 (ii) If *f* is strongly neutrosophic *grb***-**continuous, then *f* is neutrosophic continuous.

Proof. (i) Let $f: X \to Y$ be perfectly neutrosophic *grb***-**continuous. Let *V* be a a neutrosophic open set in *Y*. Then *V* is neutrosophic *grb***-**open set in *Y*. Since *f* is perfectly neutrosophic \mathcal{G} *zb***-**continuous, $f^{-1}(V)$ is neutrosophic clopen in *X*. Hence *f* is perfectly neutrosophic continuous.

(ii) Let $f: X \to Y$ be strongly neutrosophic grb-continuous. Let G be aneutrosophic open set in *Y*. Then *G* is neutrosophic *grb***-**open set in *Y*. Since *f* is strongly neutrosophic *grb***-**continuous, $f^{-1}(G)$ is neutrosophic open in X. Therefore f is neutrosophic continuous.

Theorem 6.4. Let $f: X \to Y$ be strongly neutrosophic *grb***-**continuous and *A* be a neutrosophic open set in *Y*. Then the restriction map, $f_A : A \rightarrow Y$ is strongly neutrosophic *grb***-**continuous.

Proof. Let *V* be any neutrosophic *grb***-**open set in *Y*. Since f is strongly neutrosophic grb **-continuous,** $f^{-1}(V)$ is neutrosophic open in X. But $f_A^{-1}(V) = A \cap f^{-1}(V)$. Since A and $f^{-1}(V)$ are neutrosophic open, $f_A^{-1}(V)$ is neutrosophic open in *A*. Hence *A f* is strongly neutrosophic *grb***-**continuous.

Theorem 6.5. Every perfectly neutrosophic grb-continuous mapping $f:(X,T_N) \to (Y,\sigma_N)$ is strongly neutrosophic *grb***-**continuous.

Proof. Let $f: X \rightarrow Y$ be perfectly neutrosophic grb-continuous and *V* be neutrosophic grb-open set in Y . Since f is perfectly neutrosophic \mathcal{G} *zb***-**continuous, $f^{-1}(V)$ is neutrosophic clopen in *X*. That is both neutrosophic open and neutrosophic closed in X . Hence f is strongly neutrosophic *grb***-**continuous.

Theorem 6.6. If $f: (X, T_N) \to (Y, \sigma_N)$ and and $g:(Y,\sigma_{N})\to (Z,\eta_{N})$ are strongly neutrosophic grb-continuous, then $gof:(X,T_N)\to (Z,\eta_N)$ is also strongly neutrosophic *grb***-**continuous.

Proof. Let *V* be a neutrosophic open set in *Z*. Since *g* is a strongly neutrosophic α *grb***-**continuous mapping, $g^{-1}(V)$ is neutrosophic open in *Y*. Then $g^{-1}(V)$ is neutrosophic open in *Y*. Since *f* is a strongly neutrosophic *grb***-**continuous mapping, $f^{-1}[g^{-1}(V)] = (g \circ f)^{-1}(V)$ is neutrosophic open in *X*. Therefore, $g \circ f$ is strongly neutrosophic *grb***-**continuous.

Theorem 6.7. If $f:(X, T_N) \to (Y, \sigma_N)$ and and $g:(Y,\sigma_{N})\to (Z,\eta_{N})$ are perfectly neutrosophic *grb***-**continuous mappings, then their composition $gof: (X, T_N) \rightarrow (Z, \eta_N)$ is also perfectly neutrosophic *grb***-**continuous mapping.

Proof. Let *V* be a neutrosophic *grb***-**open set in *Z*. Since *g* is a perfectly neutrosophic \mathcal{G} ² \mathcal{G} **-**continuous mapping, $g^{-1}(V)$ is neutrosophic clopen in *Y*. That is $g^{-1}(V)$ both neutrosophic open and neutrosophic closed in X. Then $g^{-1}(V)$ is neutrosophic *grb***-**open set in *X*. Since *f* is a perfectly neutrosophic *grb***-**continuous mapping, $f^{-1} [g^{-1}(V)] = (g \circ f)^{-1}(V)$ is neutrosophic clopen in X . Therefore $g \circ f$ is perfectly neutrosophic *grb***-**continuous.

Theorem 6.8. Let $f:(X, T_N) \to (Y, \sigma_N)$ and and $g:(Y,\sigma_{N})\to (Z,\eta_{N})$ be mappings. Then the following statements are true.

 (i) If *g* is strongly neutrosophic *grb***-**continuous and *f* is neutrosophic *grb***-**continuous, then *g f* is neutrosophic *grb***-**irresolute.

 ii If *g* is perfectly neutrosophic *grb***-**continuous and f is neutrosophic continuous, then $g \circ f$ is strongly neutrosophic *grb***-**continuous.

(iii) If g is strongly neutrosophic g *rb*-continuous and *f* is perfectly neutrosophic *grb* **-**continuous, then $g \circ f$ is perfectly neutrosophic *grb***-**continuous.

(iv) If g is neutrosophic g *rb*-continuous and f is strongly neutrosophic *grb*-continuous, then *gof* is neutrosophic continuous.

Proof. (i) Let V be a neutrosophic gzb-open set in *Z*. Since *g* is a strongly neutrosophic *grb* **-**continuous mapping, $g^{-1}(V)$ is neutrosophic open set in Y . Since f is a neutrosophic *grb***-**continuous mapping,

gno-continuous mapping,
 $f^{-1}\left[g^{-1}(V)\right] = (gof)^{-1}(V)$ is neutrosophic α *grb***-**open set in *X*. Hence α *gof* is neutrosophic σ *zb*-irresolute. (ii) Let *V* be a neutrosophic *grb***-**open set in *Z*. Since *g* is a perfectly neutrosophic *grb*-continuomapping, $g^{-1}(V)$ is neutrosophic clopen set in *Y*. That is, $g^{-1}(V)$ is both neutrosophic open and neutrosophic closed. Since f is a neutrosophic g *rb*-continuous mapping, $f^{-1}[g^{-1}(V)] = (g \circ f)^{-1}(V)$ is neutrosophic open in *X*. Therefore *gof* is strongly neutrosophic *grb***-**continuous.

(iii) Let V be a neutrosophic ϕ *rb*-open set in Z. Since *g* is a strongly neutrosophic *grb* **-**continuous mapping, $g^{-1}(V)$ is neutrosophic open set in *Y*. Since every neutrosophic open set is neutrosophic *grb*-open set. So $g^{-1}(V)$ is neutrosophic *grb***-**open set in *X*. Since *f* is a perfectly neutrosophic *grb***-**continuous mapping, $f^{-1} [g^{-1}(V)] = (g \circ f)^{-1}(V)$ is neutrosophic clopen in X . Hence $g \circ f$ is perfectly neutrosophic *grb***-**continuous.

 iv Let *V* be a neutrosophic open set in *Z*. Since *g* is a neutrosophic *grb***-**continuous mapping, $g^{-1}(V)$ is neutrosophic grb-open set in Y. Since *f* is a strongly neutrosophic *grb***-**continuous map, f is a strongly neutrosophic *g*_L continuous map,
 $f^{-1}\left[g^{-1}(V)\right] = (gof)^{-1}(V)$ is neutrosophic open in X . So $g \circ f$ is neutrosophic continuous.

VII. NEUTROSOPHIC CONTRA grb-CONTINUOUS MAPPINGS AND NEUTROSOPHIC CONTRA grb-IRRESOLUTE MAPPINGS

In this section, we introduce the concepts of neutrosophic contra *grb***-**continuous mappings and neutrosophic contra *grb***-**irresolute mappings and investigate their fundamental properties and characterizations.

Definition 7.1. A mapping $f: (X, T_N) \to (Y, \sigma_N)$ is said to be neutrosophic contra-continuous if the inverse image of every neutrosophic open set in *Y* neutrosophic closed set in *X*.

Definition 7.2. A mapping $f: (X, T_N) \to (Y, \sigma_N)$ is called neutrosophic contra *grb***-**continuous if the inverse image of every neutrosophic open set in *Y* neutrosophic *grb***-**closed in *X*.

Theorem 7.3. Let $f:(X,T_N) \to (Y,\sigma_N)$ be a neutrosophic contra -continuous mapping. Then *f* is neutrosophic contra *grb***-**continuous.

Proof. Let *V* be any neutrosophic open set in *Y*. Since f is neutrosophic contra continuous, $f^{-1}(V)$ is neutrosophic closed set in *X*. As every neutrosophic closed set is neutrosophic grb-closed, we have $f^{-1}(V)$ is neutrosophic g _z b **-**closed set in *X*. Therefore *f* is neutrosophic contra *grb***-**continuous.

Theorem 7.4. A mapping $f:(X,T_N) \rightarrow (Y,\sigma_N)$ is neutrosophic contra *grb***-**continuous if and only if the inverse image of every neutrosophic closed set in *Y* is neutrosophic *grb***-**open set in *X*.

Proof. Let *V* ba a neutrosophic closed set in *Y*. Then V^c is neutrosophic open set in *Y*. Since *f* is neutrosophic contra σ *zb***-**continuous, $f^{-1}(V^c)$ is neutrosophic *grb***-**closed set in *X*. But $1 (V^c) = 1 - f^{-1} (V)$ $f^{-1}(V^c) = 1 - f^{-1}(V)$ and so $f^{-1}(V)$ is neutrosophic *grb***-**open set in *X*. Conversely, assume that the inverse image of every neutrosophic closed set in *Y* is neutrosophic *grb***-**open in *X*. Let W be a neutrosophic open set in Y. Then W^C is

neutrosophic closed in *Y*. By hypothesis utrosophic closed in
 $1(W^c) = 1 - f^{-1}(W)$ is i $f^{-1}(W^c) = 1 - f^{-1}(W)$ is neutrosophic grb-open in *X*, and $f^{-1}(W)$ is neutrosophic grb **-closed set in** *X*. Thus *f* is neutrosophic contra

Theorem 7.5. If a mapping $f: (X, T_N) \to (Y, \sigma_N)$ is neutrosophic contra grb-continuous and $g:(Y,\sigma_N) \to (Z,\eta_N)$ is neutrosophic continuous, then their composition neutrosophic continuous, then their composition
 $g \circ f : (X,T_N) \to (Z,\eta_N)$ is neutrosophic contra *grb***-**continuous.

Proof. Let *W* ba a neutrosophic open set in *Z*. Since *g* is neutrosophic continuous, $g^{-1}(W)$ is neutrosophic open set in *Y*. Since *f* is neutrosophic contra *grb***-**continuous, $f^{-1}\left[g^{-1}(W)\right]$ is neutrosophic gzb-closed set in X. But $(gof)^{-1}(W) = f^{-1}[g^{-1}(W)]$. Thus gof is neutrosophic contra *grb***-**continuous.

Definition 7.6. A mapping $f:(X,T_N) \rightarrow (Y,\sigma_N)$ is called neutrosophic contra *grb***-**irresolute if the inverse image of every neutrosophic *grb***-**open set in *Y* is neutrosophic *grb***-**closed in *X*.

Theorem 7.7. If a mapping $f: (X, T_{N}) \rightarrow (Y, \sigma_{N})$ is neutrosophic contra *grb***-**irresolute, then it is neutrosophic contra *grb***-**continuous.

Proof. Let *V* be a neutrosophic open set in *Y*. Since every neutrosophic open set is neutrosophic *grb***-**open, *V* is neutrosophic *grb***-**open set in *Y*. Since *f* is neutrosophic contra *grb***-**irresolute, $f^{-1}(V)$ is neutrosophic gnb-closed set in X. Thus *f* is neutrosophic contra *grb***-**continuous.

Theorem 7.8. Let (X, T_N) , (Y, σ_N) and (Z, η_N) be neutrosophic topological spaces. If $f: (X, T_N) \to (Y, \sigma_N)$ is neutrosophic contra grb-irresolute and $g:(Y,\sigma_N) \to (Z,\eta_N)$ i is neutrosophic *grb***-**continuous, then $g \circ f : (X, T_N) \rightarrow (Z, \eta_N)$ is neutrosophic contra *grb***-**continuous.

Proof. Let *W* be any neutrosophic open set in *Z*. Since *g* is neutrosophic *grb***-**continuous, $g^{-1}(W)$ is neutrosophic gxb-open set in Y. Since *f* is neutrosophic contra *grb***-**irresolute, $f^{-1} \left[g^{-1}(W) \right]$ is neutrosophic grb-closed set in *X*. But $(gof)^{-1}(W) = f^{-1}[g^{-1}(W)]$. Thus gof is neutrosophic contra *grb***-**continuous.

Theorem 7.9. If $f: (X, T_N) \to (Y, \sigma_N)$ is neutrosophic *grb***-**irresolute and

 $g:(Y,\sigma_{N})\rightarrow (Z,\eta_{N})$ is neutrosophic contra *grb***-**irresolute, then their composition gLD-intesolute, then their composition
gof: $(X, T_N) \rightarrow (Z, \eta_N)$ is neutrosophic contra *grb***-**irresolute mapping.

Proof. Let *W* be any neutrosophic *grb***-**open set in *Z*. Since *g* is neutrosophic contra g *zb***-**irresolute, $g^{-1}(W)$ is neutrosophic *grb***-**closed set in *Y*. Since *f* is neutrosophic grb-irresolute, $f^{-1}[g^{-1}(W)]$ is neutrosophic *grb***-**closed set in *X*. But gxb-closed set in
 $(g \circ f)^{-1}(W) = f^{-1}[g^{-1}(W)].$ Thus *g*f is neutrosophic contra *grb***-**irresolute.

VIII. CONCLUSION

In this research article, we have introduced and studied the properties of neutrosophic *grb***-**continuous functions, neutrosophic *grb***-**irresolute functions, neutrosophic *grb***-**closed functions, neutrosophic *grb***-**open functions, strongly neutrosophic *grb***-**continuous functions, perfectly neutrosophic *grb***-**continuous functions, neutrosophic contra *grb***-**continuous functions, and neutrosophic contra *grb***-**irresolute functions in neutrosophic topological spaces and established the relations between them. We have obtained fundamental characterizations of theses mappings and investigated preservation properties. We expect the results in this chapter will be basis for further applications of mappings in neutrosophic topological spaces. **ACKNOWLEDGMENT**

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