Integration of migration flows. A diffusive theory

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***Abstract*—The subject of this research is the presentation of a model for studying the integration of migration flows with the resident population. The basic element for social cohesion is the cultural level of the people involved. In this study, we hypothesize a similarity between diffusion laws of the heat and culture, represented respectively by equations on the knowledge and temperature. The integration of migration flows is described by the use of the Cahn-Hilliard equation.**

Keywords: Social and economic integration, Thermodynamics, Phase transitions, Cahn-Hilliard equation.

I. Introduction

Fig.1 - Symbolic integration pictures

Fig.2 - An impressive picture that well describes migration issues

The growing role of the problems linked to migration and integration for the political and economic development of social systems is becoming very important. Stimulating interest in the mathematical modeling of migration and integration.

In this framework, the equations which describe the evolution of the culture is represented by diffusion equations on the knowledge, likewise to the equation that describes the heat diffusion.

Moreover, we suppose that the social cohesion in the integration processes is controlled by the cultural level of the populations [6],[5],[4],[7],[1],[2],[3],[9],[10].

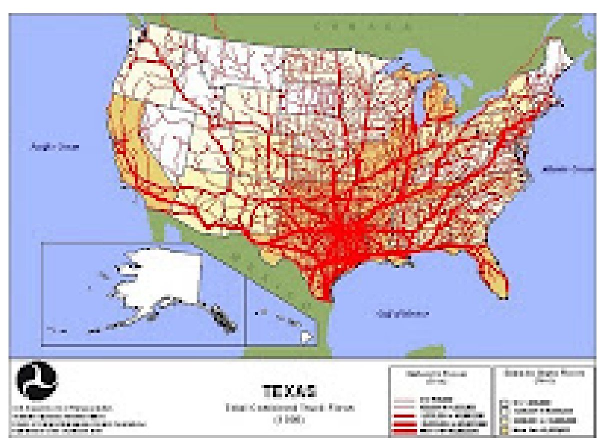

Fig.3 - The flow migration form Texas to US

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In this work we present a summary of the paper [8], in which it was studied the migration and integration of different populations by a differential system containing the Cahn-Hilliard equation. This issue describes the integration or separation law of two ethnic fluxes, according to a control factor given by the cultural levels of two populations, whose evolutions are described by a system of diffusion equations. Moreover, we assume that the homogenization process occurs when the mean of two cultural levels exceeds a critical value.

Fig.4 Diffusion process

II. INTEGRATION MODEL

In this section, we study the interaction of two ethnic groups A_1 and A_2 with different cultures (traditions, religions, ecc.). So in a fixed bounded domain $\Omega \subset \boldsymbol{R}^2$ and time interval $[0, T]$, the total mass M_1 and M_2 of two populations will be constants. In the following, we denote with ρ_1 and ρ_2 the local densities of the two groups, while the specific densities of the populations A_1 and A_2 are the same, denoted by ρ. Finally, the local concentration $c \in [-1, 1]$ of the component A_1 is given by $c = \frac{2\rho_1 - \rho}{\rho}$. In this framework, we consider the mean velocity $\mathbf{v}$ of the mixture, defined by

$$\mathbf{v} = \frac{\rho_1 \mathbf{v}_1 + \rho_2 \mathbf{v}_2}{\rho} \tag{II.1}$$

where $\mathbf{v}_1$ and $\mathbf{v}_2$ are the velocity related with the components A_1 and A_2. Thus, the evolution of the mixture will be represented by the equation of a viscous incompressible fluid

$$\rho \dot{\mathbf{v}} = -\nabla p + \rho \nabla \cdot (\nabla c \otimes \nabla c) + \nabla \cdot \upsilon(c) \nabla \mathbf{v} + \rho \mathbf{b} \tag{II.2}$$

where p is the pressure, $\upsilon(c)$ is the viscosity coefficient depending on c, the vector $\mathbf{b}$ denotes the external body forces. Moreover, because the density of the mixture is constant., we have $\nabla \cdot \mathbf{v} = 0$

Fig.5 - The homogenization of the mixture

Fig.6 - Integration process

The behavior of the concentration c will be described by the Cahn-Hilliard equation

$$\rho \dot{c} = \nabla \cdot M(c) \nabla \mu \tag{II.3}$$

where $M(c)$ is the mobility such that $M(c) \geq 0$, $M(-1) = M(1) = 0$. While μ is the supplemented potential defined by

$$\mu(c, \varphi_1, \varphi_2) = \gamma \nabla^2 c - \varphi_0 H'(c) - \frac{\varphi_1 + \varphi_2}{2} L'(c) \tag{II.4}$$

where the potentials H and L are defined by

$$H(c) = \frac{1}{4}(c^2 - 1)^2 \quad , \quad L(c) = \frac{c^2}{2} \tag{II.5}$$

while $\varphi_1 > 0$ and $\varphi_2 > 0$ represent the knowledge levels of the two components, while φ_0 is a critical value, which denotes the integration-separation phase transition, controlled by the mean value $\frac{\varphi_1 + \varphi_2}{2}$. Hence, we obtain by (II.3) and (II.4) the equation on the concentration c

$$\rho \dot{c} = \nabla \cdot M(c) \nabla \left(\gamma \nabla^2 c - \varphi_0 H'(c) - \frac{\varphi_1 + \varphi_2}{2} L'(c)\right) \tag{II.6}$$

Finally, for this problem we consider the boundary conditions

$$M(c) \nabla \mu \cdot \mathbf{n} = 0 \quad , \quad at \ \ \partial\Omega \tag{II.7}$$

where $\mathbf{n}$ is the unit outward normal. Then, we study the evolution of the educational level by a diffusion equation. Thus, we introduce two equations related with the knowledge, which describe the cultural balance laws of two ethnic groups

$$\rho \dot{\varphi}_1 - \frac{1}{2}\dot{L}(c) + \frac{\nu}{2}\nabla^2 \mathbf{v} = \nabla \cdot \delta_1 \nabla \varphi_1 - \alpha(\varphi_1 - \varphi_2) + \rho s_1 \tag{II.8}$$

$$\rho \dot{\varphi}_2 - \frac{1}{2}\dot{L}(c) + \frac{\nu}{2}\nabla^2 \mathbf{v} = \nabla \cdot \delta_2 \nabla \varphi_2 - \alpha(\varphi_2 - \varphi_1) + \rho s_2 \tag{II.9}$$

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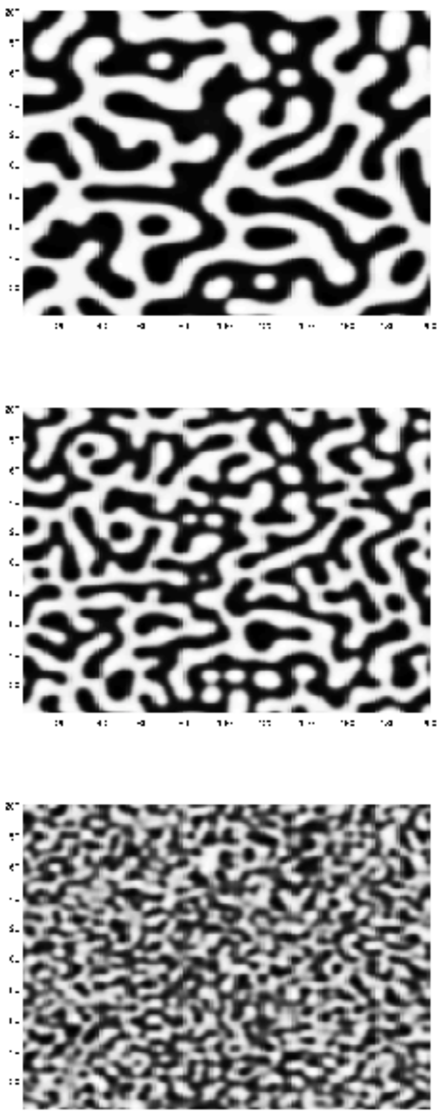
where $\alpha \geq 0$, s_1 and s_2 represent the cultural supplies, while δ_1 and δ_2 are the cultural conductivity (diffusibility) of the component A_1 and A_2. Therefore, the differential system is given by the equations (II.2), (II.6), (II.8) and (II.9) with the boundaries conditions (II.7) and

$$\begin{aligned} \mathbf{v}(x,t)|_{\partial\Omega} &= 0\ ,\ \ \nabla c(x,t)\cdot\mathbf{n}(x)|_{\partial\Omega} = 0\ , \\ \nabla\varphi_1(x,t)\cdot\mathbf{n}(x)|_{\partial\Omega} &= 0\ ,\ \ \nabla\varphi_2(x,t)\cdot\mathbf{n}(x)|_{\partial\Omega} = 0 \end{aligned} \tag{II.10}$$

and the initial conditions

$$\begin{aligned} \mathbf{v}(x,0) &= \mathbf{v}_0(x)\ ,\ c(x,0) = c_0(x)\ \ ,\ \ x\in\Omega \\ \varphi_1(x,0) &= \varphi_{10}(x)\ ,\ \varphi_2(x,0) = \varphi_{20}(x)\ \ ,\ \ x\in\Omega \end{aligned} \tag{II.11}$$

The evolution of the system is well described by the following pictures.



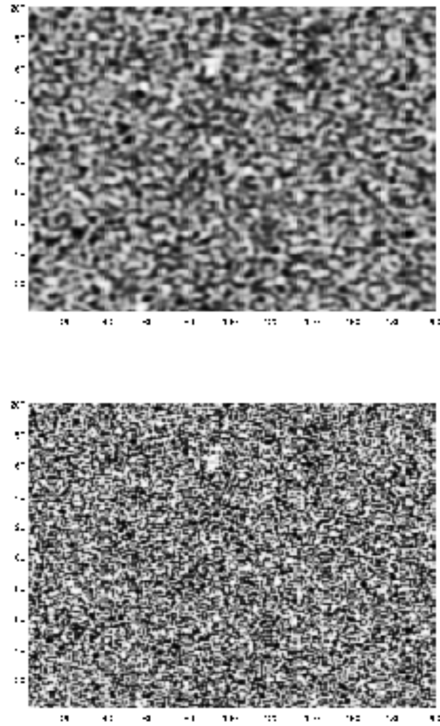


Fig.7 - The integration process by Cahn-Hilliard equation

References

[1] C. Argyris and D. Schon, Organizational Learning: a Theory of Action Perspective, Reading, Menlo.Park:Addison-Wesley,1978.

[2] A. Berti, I. Bochicchio, *A mathematical model for phase separation: A generalized Cahn-Hilliard equation*, Math.Meth.Appl.Sci. (2011).

[3] L. Bevilacqua, A. C. Galeão, F. Pietrobon-Costa, S. L. Monteiro. Knowledge diffusion paths in a research chain. Mecanica Computacional. Vol. 24, 2061-2069. 2010.

[4] J.W. Cahn, C. M. Elliott and A. Novik–Cohen, *The Cahn–Hilliard equation with a concentration dependent mobility: motion of minus Laplacian of the mean curvature,* European J. Appl. Math. 7, No.3, 287-301.

[5] L.L. Cavalli-Sforza and M. Feldman, *Cultural Transmission and Evolution,* Princeton: Princeton University Press. 1981

[6] E. Collett, Immigrant Integration in Europe in a Time of Austerity. Migration Policy Institute. Washington 2011

[7] M. Fabrizio, C. Giorgi and A. Morro, *Phase separation in quasi-incompressible Cahn–Hilliard fluids*, European Journal of Mechanics B/Fluids **30** (2011), 281—287

[8] M. Fabrizio, J. Munoz Rivera, *An integration model for two different ethnic groups*, In press on Evolution Equations and Control Theory.

[9] M.E. Gurtin, *Generalized Ginzburg-Landau and Cahn-Hilliard equations based on a microforce balance*, Physica D **92** (1996), 178-192.

[10] G. Hedlund, *A model of knowledge management and the N-Form corporation*, Strategic Management Journal, **15**, 73-90,1994